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# Revisited water-oriented relationships between a set of farmers and an aquifer: accounting for lag effect

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**Abstract**

Many environmental problems are due to damages caused by stock of pollutants which accumulate with time lag to their emission. In this paper, we focus on nitrates used in agriculture which can pollute groundwater years after their initial use. A dynamic optimal control problem with heterogeneous farmers is proposed. Usual structural parameters like the discount rate, the natural clearing rate, the lagged time interval between the soil-level pollution occurrence and the impact on groundwater are taken into account. We also examine pollution as caused by a continuous set of farms characterized by their individual performance index and by their individual marginal contribution to the pollution. The issue is further investigated by taking account of change in the information context, successively related to perfect information and to asymmetric information. As a result, when the delay between the spreading of N-fertilizer and the impact on the aquifer increases, i.e., the higher the lag, the steady state pollution stock and the steady state shadow price of the stock both increase. Moreover, asymmetric information leads to a higher stock of pollution. Given the European Union context and its directives focusing on nitrate pollution and water quality, the qualitative results provided in this paper should help modellers and decision makers promote suitable environmental policies.

Key-words : farming pollution, aquifer, nitrate, time lag, optimal control, mechanism design

Code JEL : Q25, C61, D62, D82

# 1 Introduction

Many environmental problems are due to damages caused by stocks of pollutants which accumulate with time lag to their emission. For example, nitrates used in agriculture can pollute aquifers years after their initial use. This pollution problem is non negligible because aquifers provide most of the drinking water in the world (25 to 45% in France) [1] and they are very vulnerable to surface pollution. It is mainly due to the nitrates from overapplying of N-fertilizer and spreading manure in agriculture. When the nitrates are ingested in too large quantities, they have a toxic effect on human health and contribute to eutrophication. Moreover, the effects of nitrates are visible 30-60 years after their use. The purpose of this article is to study the lag effect in accumulation of pollution stock in an optimal control model, and whether this lag effect is amplified or not by asymmetric information when the social planner is not informed about individual farm characteristics.

In the economic literature, the lag has been firstly introduced into the accumulation of capital (Rustichini (1989), Asea and Zak (1999) ), and more recently into the pollution stock (Brandt-Pollmann et al. (2008), Winkler (2010)). Winkler (2010) analyzes a generic optimal control model with one control that accumulates to a stock with a fixed delay. He shows that the optimal paths are generally oscillatory, but monotonic when the objective function is additively separable in the stock and the control. However, our basic problem does not match the case of oscillatory paths. We assume a separable objective function which allows to solve the lag problem in the different information contexts. Despite this separability assumption, to our knowledge, literature does not deliver how accounting for lag effect in pollution stock can modify the policy of the social planner. *A fortiori* the problem still remains open in case of asymmetric information.

[1] http://www.cnrs.fr/cw/dossiers/doseau/decouv/degradation/07_pollution.htm

In this study, we are dealing with a set of farmers involved in the pollution of an aquifer by nitrates. The regulation of agricultural non point source pollution is complex: farmers work under various production conditions (climate, soils, etc .). Even in the case of the same technique and the same input level used by two farmers, the consequences on the pollution level can be different. In our approach, the pollution is caused by a continuous set of farms characterized by their individual performance index and by their individual marginal contribution to the pollution. This analysis is in line with studies carried out by Dasgupta et al. (1980) and Laffont and Tirole (1986). We propose a dynamic optimal control problem with heterogeneous farmers, in which emissions accumulate with a time lag to a pollution stock.

As a result, the lag acts for increasing the stock and its shadow price at the steady state. This effect is augmented when asymmetric information occurs between the 'informed" farmers and the social planner.

The paper is organized as follows. Section 2 is devoted to the presentation of the basic model. In section 3, we set out the generic control problem with time lagged stock accumulation when the social planner is completely informed about individual farm characteristics. In section 4, we develop the analysis of the optimal control problem when asymmetric information drives the mechanism design which should be implemented by the regulator. Finally in section 5, we illustrate differences between accounting for or not time lag. We also compare results in case of perfect information to results in case of asymmetric information.

# 2 Basic elements

Let us consider the set of farmers contributing to the nitrate pollution of an aquifer. Farming activity is represented by the demand of $N$-fertilizer which is denoted by $x$. Activity depends on performance characteristics summarized by the one-dimensional $\theta$ parameter. The individual farm profit is represented by the function $\pi(x, \theta)$, in which the farm performance characteristics $\theta$ is spread over the interval $\Theta = [\underline{\theta}, \bar{\theta}]$.The likelihood density is denoted by $\gamma(\theta)$ and assumed to be strictly positive at any point within the interval :

$$\gamma(\theta) > 0 \quad \forall\theta \tag{H1}$$

The related cumulative function is denoted by $\Gamma(\theta)$. It should be noted that "performance" is farm-dependent rather than farmer-dependent. In our case, this performance refers to soil quality more than to farmer' ability. In other words, when asymmetric information on farm characteristics comes in our analysis, we face an adverse selection problem.

The $\pi$ function is assumed to be twice continuously differentiable. The usual assumption of decreasing return to scale and the assumed positive marginal profit when $x$ is close to 0 hold here :

$$\pi_{xx} < 0 \tag{H2}$$

$$\pi_x(0, \theta) > 0 \quad \forall\theta \tag{H3}$$

We assume that the marginal profit variation regarding the $\theta$ characteristics keeps the same sign. We choose the positive sign, so the marginal profit increases when the $\theta$ characteristics increases :

$$\pi_{x\theta} > 0 \tag{H4}$$

Regarding the marginal farm profit and further formal analysis coming further in the paper, let us consider the $x$-variable equation $\pi_x(x, \theta) = c$. Hypotheses (H2) and (H4) immediately deliver the solution $x = \phi(\theta, c)$ as a function of the performance index $\theta$ and the $x$-based tax $c$ changes :

$$\pi_x(\theta, \phi(\theta, c)) = c \quad \Rightarrow \quad \phi_\theta > 0 \;\; \text{and} \;\; \phi_c < 0 \tag{R1}$$

In other words, the factor demand increases when the performance parameter increases, and the factor demand decreases when the apparent $x$-tax set on the factor increases.

The farming activity is assumed to occur over time. At any time $t$, the $\theta$-farm use of $x$ leads to an increase in the global farming profit by $\pi(x(\theta,t),\theta)$ (meaning no change in prices). Accordingly, the time-unit global profit is expressed by $\int_\Theta \pi(x(\theta,t),\theta)\gamma(\theta)d\theta$.

Regarding the environmental impact and related damage, we can start by applying a standard framework. The state of our aquifer system is characterized by the nitrate stock per volume unit and denoted by $z$. The dynamic evolution over time is the result of a double-side effect. On one hand, the clearing effect takes the form of an usual exponential decline characterized by the decline rate $\tau$. On the other hand, the amount of $N$-fertilizer consumed by the $\theta$ farm additively contributes to increase the pollution. The marginal contribution related to $x$ depends on $\theta$ and the time-unit contribution of the $\theta$ farm is $a(\theta)\ x(\theta,t)$.

However, a slight difficulty arises when we introduce the lag effect of $N$-fertilizer use on the nitrate concentration in the aquifer. We denote the lag parameter (not depending on $\theta$) by $\beta$. The time evolution of the environmental systems is described by the equation :

$$\dot{z}(t) = -\tau z(t) + \int_\Theta a(\theta)x(\theta,t-\beta)\gamma(\theta)d\theta \tag{1}$$

Expecting that the regulatory body will be asked to design the optimal individual farm demand for the input $x(\theta,t)$ at time 0 for any further time $t$, we assume that the body integrates knowledge related to the initial state of the aquifer and to the short past farming activity. In addition, we recall that the input has to be non negative. This is expressed by the following assumption :

$$z(0) = z_0\ ;\ \ x(\theta,t) = \epsilon(\theta,t)\ \ \forall\theta\in\Theta\ \ \forall t\in[-\beta,0[\ ;\ \ x(\theta,t)\geq 0\ \forall\theta\in\Theta,\forall t\geq 0 \tag{H5}$$

The time unit damage function is expressed by the twice differentiable function depending on $z$ and denoted by $D(z)$. The assumptions related to the damage function are :

$$D_z(0) = 0 \text{ and } (D_z > 0 \ \forall z > 0) \tag{H6}$$

$$D_{zz} > 0 \tag{H7}$$

Let us notice that (H6) and (H7) leads to $D_z > 0 \ \forall z > 0$.

Finally, the discount rate is denoted by $\delta$, and the marginal cost of public funds is denoted by $\rho$. This last parameter enters the analysis when contractual incentives are taken into consideration.

The economic analysis that follows is based on a partial equilibrium approach with no price feedbacks from the rest of the economy.

# 3 Long run optimal trade-off between production and pollution in the case of complete information

When information upon farmers is complete, the social planner's objective is :

$$W = \int_0^{\infty} [\int_{\Theta} \pi(x(\theta,t),\theta)\gamma(\theta)d\theta - D(z(t))]e^{-\delta t}dt \tag{2}$$

Accordingly, this programme is expressed below :

$$\max_{x(\theta,t),z(t)} W \ \text{ subject to (1), } (H5) \tag{3}$$

Differently from the usual optimal control programme, the lag term appearing in the state dynamics (1) does not allow us to directly apply the Pontryagin theorem. The solution arises when we consider the transformation of the command variable $y(\theta,t) = x(\theta, t-\beta)$. The objective function and the state evolution equation are transformed as follows :

$$W=-\int_{-\beta}^{0}\int_{\Theta}\pi(x(\theta,t),\theta)\gamma(\theta)d\theta e^{-\delta t}dt+\int_{0}^{\infty}[e^{\delta\beta}\int_{\Theta}\pi(y(\theta,t),\theta)\gamma(\theta)d\theta-D(z(t))]e^{-\delta t}dt \tag{4}$$

$$\dot{z}(t)=-\tau z(t)+\int_{\Theta}a(\theta)y(\theta,t)\gamma(\theta)d\theta \tag{5}$$

Thanks to the (H5) assumption, the first integral component of this last $W$ expression can be taken out of the programme. Aiming at the use of the maximum principle, we define the current-value Hamiltonian in which the shadow price of the pollution stock denoted by $\lambda(t)$ and is designed as to take a positive value :

$$H^{c}=e^{\delta\beta}\int_{\Theta}\pi(y(\theta,t),\theta)\gamma(\theta)d\theta-D(z(t))-\lambda(t)[\int_{\Theta}a(\theta)y(\theta,t)\gamma(\theta)d\theta-\tau z(t)] \tag{6}$$

According to our technical assumptions, the Pontryagin theorem delivers the conditions holding the optimal solution : $\{y^*(\theta,t),z^*(t),\lambda^*(t)\}$ :

$$y^*(\theta,t) \text{ maximizes } H^c(y,z^*,\lambda^*) \tag{7}$$

$$\dot{\lambda}^*(t)-\tau\lambda^*(t)=H^c_z(y^*,z^*,\lambda^*) \tag{8}$$

Our "convex" problem leads to the following equations :

$$\pi_x(y^*(\theta,t),\theta)=a(\theta)\lambda^*(t)e^{-}\delta\beta \tag{9}$$

$$\dot{\lambda}^*(t)-(\tau+\delta)\lambda^*(t)=D'(z^*(t)) \tag{10}$$

The transversality condition enters the complete system of conditions :

$$\lim_{t\to\infty}\lambda(t)e^{-\delta t}z(t)=0 \tag{11}$$

Condition (9) expresses that the $\theta$ farmer' profit provided by one additional unit of polluting input equals the discounted cost of the related marginal pollution evaluated at time $t+\beta$ and weighted by the individual polluting contribution $a(\theta)$. The $y$ solution of this equation arises through the relation (R1) $y^*(\theta,t) = \phi(\theta, a(\theta)\lambda(t))$. The complete solution of the Regulator's program is provided by the implicit relation between the command $x$ and the shadow price $\lambda$, and by the two-dimension differential system, as summarized by the equation set (R2) :

$$
\begin{aligned}
&\forall\theta,\ \forall t>0\ :\ x^*(\theta,t) = \phi(\theta, a(\theta)\lambda(t+\beta)e^{-\delta\beta}) \\
&\forall\theta\in\Theta,\ \forall t\in[-\beta,0[\ : x^*(\theta,t) = \epsilon(\theta,t) \\
&\dot{z}^*(t) = -\tau z^*(t) + \textstyle\int_\Theta a(\theta)x^*(\theta,t-\beta)\gamma(\theta)d\theta \\
&\dot{\lambda}^*(t) - (\tau+\delta)\lambda^*(t) = -D'(z^*(t)) \\
&z^*(0) = z_0\ \ ;\text{ the tranversality condition satisfied}
\end{aligned}
\tag{R2}
$$

There is only one steady state related to this system (proof in appendix A). The technical assumptions described above lead to deliver too a graphics describing the paths related to this differential system.

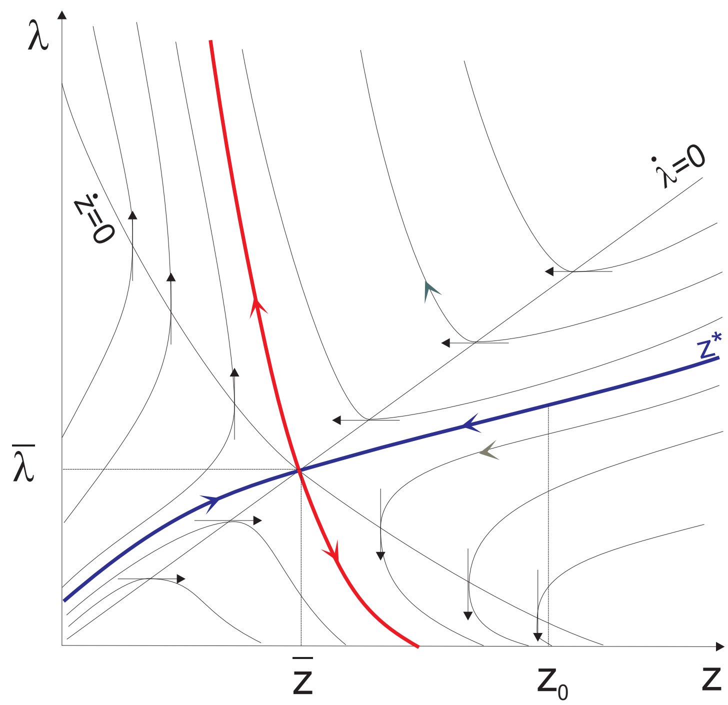


Figure 1: Phase diagram describing the paths linking the pollution state $z$ and its shadow price $\lambda$.

Let us focus on the steady state $(\bar{z}, \bar{\lambda})$ defined by $\{\dot{y} = 0;\ \dot{z} = 0\}$. We are interested by the impacts of the parameters $\beta$, $\delta$, $\tau$ on the steady state, leading us to summarize results in propositions 3.1, 3.2 and 3.3.

**Proposition 3.1** *When the delay is increased between the spreading of $N-$fertilizer on the farm and the impact of it, i.e., the higher the lag, the greater the increase in the pollution level and the higher the shadow price in the steady state.*

**Proposition 3.2** *When the discount rate increases, the steady state pollution level and the steady state shadow price both increase.*

**Proposition 3.3** *When the decline rate increases, i.e more nitrates are absorbed by the aquifer the steady state pollution level and the steady state shadow price both increase.*

Proofs are delivered in appendix B.

Having in mind the contractual approach which supports the analysis of the asymmetric information problem (see the section 4), we introduce the Regulator's choice in supplying contracts to any $\theta$ farm. A contract is characterized by a two dimensions function $(q(\theta, t), s(\theta, t))$ in which $q$ refers to the upper limit of $x$-use of polluting input and $s$ refers to the individual fund transfer as the counterpart of profit decrease. Contracts are designed to be freely accepted by the farms, consequently the Regulator has to prevent farmers from refusing the contracts when their participation is expected as socially beneficial.

The transfers call for costly public funds (i.e. one budget unit costs $1 + \rho$) and the social objective is now expressed like :

$$W = \int_0^{\infty} \{\int_{\Theta} [\pi(q(\theta, t), \theta) - \rho s(\theta, t)]\gamma(\theta) d\theta - D(z(t))\} e^{-\delta t} dt \qquad (12)$$

In the complete information case, there is no place for informational rent. The reservation utility of the $\theta$ farm is the unconstrained profit characterized by the $q$-consumption equal to $\phi(\theta, 0)$ (constant along time). When public funds are costly the individual discounted transfer is equal to the individual profit variation :

$$\int_0^\infty s(\theta, t)e^{-\delta t}dt = \int_0^\infty [\pi(\phi(\theta, 0), \theta)\pi(q(\theta, t), \theta)]e^{-\delta t}dt \tag{13}$$

The public objective can be rewritten by substitution of the transfer expressed above, so that the Regulator's programme is now :

$$\max_{q(.,.)} W = \int_0^\infty \{\int_\Theta [(1+\rho)\pi(q(\theta, t), \theta) - \rho\pi(\phi(\theta, 0), \theta)]\gamma(\theta)d\theta - D(z(t))\}e^{-\delta t}dt \tag{14}$$

arising with the unchanged dynamics of the state variable (still delivered by equation 5). The implicit solution of this programme is still provided through the change in the control variable with respect to the time lag parameter $\beta$. The contract (for any $\theta$ at any time for the quota $q$, and under an integral equation for any $\theta$-transfer $s$) and the $(z, \lambda)$ path are completely characterized by the system (R3) :

$$\begin{array}{l}
\forall\theta,\ \forall t > 0\ :\ q^*(\theta, t) = \phi(\theta, \frac{a(\theta)\lambda^*(t+\beta)e^{-\delta\beta}}{1+\rho}) \\
\forall\theta\ :\ \int_0^\infty s^*(\theta, t)e^{-\delta t}dt = \int_0^\infty [\pi(\phi(\theta, 0), \theta) - \pi(q^*(\theta, t), \theta)]e^{-\delta t}dt \\
\forall\theta \in \Theta\ \ \forall t \in [-\beta, 0[\ :\ q^*(\theta, t) = \epsilon(\theta, t) \\
\dot{z}^*(t) = -\tau z^*(t) + \int_\Theta a(\theta)q^*(\theta, t-\beta)\gamma(\theta)d\theta \\
\dot{\lambda}^*(t) - (\tau + \delta)\lambda^*(t) = -D'(z^*(t)) \\
z^*(0) = z_0\ \ \text{; the tranversality condition satisfied}
\end{array} \tag{R3}$$

When the parameter related to the shadow cost of public funds tends toward 0 (i.e. $\rho \to 0$), the system (R3) tends toward the system (R2). The non-costly transfers do not affect the solution $(q, z, \lambda)$.

The parameters $\theta$, $\lambda$ and $\delta$ have similar effects on the steady state as mentionned in the R2-analysis. Proposition 3.4 delivers the qualitative impact of the cost of public funds on the steady state (proof in appendix B).

**Proposition 3.4** *When the marginal cost of public funds is increased, the greater the increase in pollution level, the higher the shadow price in the steady state.*

# 4 The dynamic problem in the case of asymmetric information case

This section is devoted to the optimal dynamic control problem in the case of asymmetric information. In this context, the regulator has no individual information on any $\theta$ farm, but he knows the statistical distribution of $\theta$. We place our adverse selection problem in the framework of the incentive theory developed by Laffont and Tirole (1993) among others. We consider that the regulator offers a menu of contracts to any farm, and either the farmer $\theta$ selects one of the contracts or he refuses all of them. The problem of the regulator is to design the optimal menu regarding the social objective including the farm profits, the environmental damage, and the regulation costs.

The menu of contracts is a two dimension function $(q(\theta, t), s(\theta, t))$. Like in the previous complete information context, $q$ denotes the "quota" and $s$ denotes the "subsidy". Formally the regulator acts as asking any farmer at time 0 for contracting or not, and for the characteristics of his $\theta$ farm in the case of acceptance. The participating farmer selects a contract through the announce $\tilde{\theta}$. The acceptance by the farmer implies that he complies at time 0 with the upper bound $q(\tilde{\theta}, t)$ holding the $q$-input at any time $t$. He will receive the transfer $s(\tilde{\theta}, t)$.

The $\theta$ farmer's programme is to declare his optimal announce. Based on the revelation principle, the menu proposed by the regulator is a mechanism designed in such a way that the $\theta$ farmer's dominant strategy is to announce his true characteristics $\theta$. Theoretically the regulator keeps the possibility to design the menu in such a way that the optimal set of participating farmers is a subset of $\Theta$. This opportunity is explored in some papers devoted to application of the incentive theory (see Bourgeon et al. (1995)). For simplicity, we do not keep here this opportunity, even if the menu is possibly suboptimal.

Formally we consider that the functions $q$ and $s$ have the requested mathematical properties allowing to use derivatives as long as necessary. The first step of the analysis leads to characterize the incentive constraints and the participation constraint (the so-called rationality constraint). The starting point is the following $\theta$ farmer's programme which defines the farmer's optimal announce :

$$\max_{\tilde{\theta}} \int_0^\infty [\pi(q(\tilde{\theta},t),\theta) + s(\tilde{\theta},t)]e^{-\delta t}dt \tag{15}$$

Let us notice that the private discount rate is supposed to be equal to the public one $\delta$. Solving this programme using first and second order conditions, and with the help of the revelation principle, we derive incentives constraints summarized by the relations IC1 and IC2 :

$$\textstyle\int_0^\infty [\pi_x(q,\theta)\frac{\partial q}{\partial \theta} + \frac{\partial s}{\partial \theta}]e^{-\delta t}dt = 0 \tag{IC1}$$

$$\textstyle\int_0^\infty \pi_{x\theta}(q,\theta)\frac{\partial q}{\partial \theta}e^{-\delta t}dt > 0 \tag{IC2}$$

The contract is supposed to be freely accepted by the $\theta$ farmer. When the regulator aims at leading the farmer to accept the contract, he has to ensure that the farmer does not lose with the contract acceptance. The "reservation profit" of the $\theta$ farm is expressed by $\pi(\phi(\theta,0),\theta)$ (which has a constant current value along time). That leads to define the information rent $R(\theta)$ which has to be not negative as following :

$$R(\theta) = \int_0^\infty [\pi(q(\theta,t),\theta) + s(q(\theta,t),\theta) - \pi(\phi(\theta,0),\theta)]e^{-\delta t}dt \geq 0$$

Assumption H4 leads to demonstrate that the rent decreases when the $\theta$ type increases. Considering that a contract has to be accepted by any $\theta$ in $\Theta$, we can write the rationality constraint under the form (IR1) in which only the upper type $\bar{\theta}$ plays :

$$R(\bar{\theta}) = \int_0^{\infty} [\pi(q(\bar{\theta}, t), \bar{\theta}) + s(\bar{\theta}, t) - \pi(\phi(\bar{\theta}, 0), \bar{\theta})] e^{-\delta t} dt \geq 0 \qquad \text{(IR1)}$$

Let us consider the social welfare function $W$ which is now :

$$W = \int_0^{\infty} \left\{ \int_{\underline{\theta}}^{\bar{\theta}} [\pi(q(\theta, t), \theta) - \rho s(q(\theta, t), \theta)] \gamma(\theta) d\theta - D(z(t)) \right\} e^{-\delta t} dt \geq 0$$

The subsidy term is easily replaced with the help of the first order incentive condition (IC1) and integration by parts :

$$\int_0^{\infty} \int_{\underline{\theta}}^{\bar{\theta}} s(q(\theta, t), \theta) \gamma(\theta) d\theta = \int_0^{\infty} s(\bar{\theta}, t) e^{-\delta t} dt + \int_0^{\infty} \int_{\underline{\theta}}^{\bar{\theta}} \pi_x(q, t) \frac{\partial q}{\partial \theta} \Gamma(\theta) d\theta e^{-\delta t} dt$$

Like in the previous sections of the paper, regarding the state dynamic equation (1) which calls for the time lag command variable, we choose to replace the command $q(\theta, t)$ by the variable $r(\theta, t) = q(\theta, t - \beta)$ in the function $W$ :

$$\begin{aligned} W \quad = \quad & \int_0^{\infty} \left\{ e^{\delta\beta} \int_{\underline{\theta}}^{\bar{\theta}} \left[ \pi(r(\theta, t), \theta)\gamma(\theta) - \rho \pi_x(r(\theta, t), \theta) \frac{\partial r}{\partial \theta}(r(\theta, t), \theta)\Gamma(\theta) \right] d\theta - D(z(t)) \right\} e^{-\delta t} dt \\ & -\rho \int_0^{\infty} s(\bar{\theta}, t) e^{-\delta t} dt - \int_0^{\beta} \int_{\underline{\theta}}^{\bar{\theta}} \left[ \pi(r(\theta, t), \theta)\gamma(\theta) - \rho \pi_x(r(\theta, t), \theta) \frac{\partial r}{\partial \theta}(r(\theta, t), \theta)\Gamma(\theta) \right] d\theta \end{aligned}$$

In the second line of this expression, the first negative term related to the $\bar{\theta}$ subsidy weighted by $\rho$ should be as small as possible. The rationality constraint (IR1) leads to design the $\bar{\theta}$ contract in such a way that the rent $R(\bar{\theta})$ is equal to zero. The second term of the second line is explicitly computed thanks to the (H5) hypothesis ($r(\theta, t) = q(\theta, t - \beta) = \epsilon(\theta, t - \beta)$, $\forall\ t \in [0, \beta]$).

The optimal control problem of the regulator can be limited to the first line of the expression above, so that the current hamiltonian function related to the problem is :

$$H^c = e^{\delta\beta} \int_{\underline{\theta}}^{\bar{\theta}} [\pi(r, \theta)\gamma - \rho \pi_x(r, \theta) r_\theta \Gamma] d\theta - D(z) - \lambda \left[ -\tau z + \int_{\underline{\theta}}^{\bar{\theta}} a r \gamma d\theta \right]$$

The Pontryagin theorem leads to maximize the $H^c$ function according to the command $r$. Regarding the integral form of the hamiltonian as a function of $r$ and $r_\theta$, the problem can be solved by the Euler relation ($\frac{\partial H^c}{\partial r} = \frac{d}{d\theta}\frac{\partial H^c}{\partial r_\theta}$). Finally, the characterization of the full menu of contracts, the dynamic equations describing the evolution of the state $z$ and the shadow price $\lambda$ is summarized by the system R4.

The incentive mechanism at any time and for any farm is completely described by equations R4 :

$$\begin{aligned}
&\forall\theta,\ \forall t > 0\ :\ q^*(\theta,t) = \phi(\theta, \tfrac{a(\theta)\lambda^*(t+\beta)e^{-\delta\beta}}{1+\rho} - \tfrac{\rho}{1+\rho}\pi_{x\theta}(q,\theta)\tfrac{\Gamma}{\gamma})\\
&S(\bar\theta) = \textstyle\int_0^\infty s(\bar\theta,t)e^{-\delta t}dt = \int_0^\infty[\pi(\phi(\bar\theta,0),\bar\theta) - \pi(q(\bar\theta,t),\bar\theta)]e^{-\delta t}dt\\
&\forall\theta\ :\ \textstyle\int_0^\infty s(\theta,t)e^{-\delta t}dt = S(\bar\theta) + \int_0^\infty\int_\theta^{\bar\theta}\pi_x(q(u,t),u)\frac{\partial q}{\partial\theta}(u,t)du e^{-\delta t}dt\\
&\forall\theta\in\Theta\ \ \forall t\in[-\beta,0[\ :\ q^*(\theta,t) = \epsilon(\theta,t)\\
&\dot z^*(t) = -\tau z^*(t) + \textstyle\int_\Theta a(\theta)q^*(\theta,t-\beta)\gamma(\theta)d\theta\\
&\dot\lambda^*(t) - (\tau+\delta)\lambda^*(t) = -D'(z^*(t))\\
&z^*(0) = z_0\ \ \text{; the tranversality condition satisfied}
\end{aligned} \tag{R4}$$

We assume that added technical conditions referring to the (IC2) conditions hold and allow to consider that the necessary conditions delivered by the system (R4) describe the optimal solution. The optimal menu of contracts leads the regulator to design the quota $q$ for any $\theta$ at any time $t$. The subsidy appears through an integral condition.

Compared to the system R3, the steady state related to the system R4 lets an additional negative term appearing in the expression of the optimal quota ($q^* = \phi(\theta, \frac{a\lambda^* e^{-\delta\beta}}{1+\rho} - \frac{\rho}{1+\rho}\pi_{x\theta}(q,\theta)\frac{\Gamma}{\gamma})$. This additionnal term does not allow us to deliver a general result in term of lag effect. The sign of third derivatives enters the conditions which lead to the proposition 4.2. Moreover, this sign plays a crucial role in the comparison between system R3 and system R4 (proposition 4.1).

**Proposition 4.1** *In case of asymmetric information (R4), the level of pollution stock, the shadow price and the total amount of instantaneous polluting input at the steady state are higher than in case of perfect information (R3)* *when the third-derivative, $\Pi_{xx\theta}$, is negative. Otherwise, the effects are ambiguous.*

Proof is delivered in appendix C.

**Proposition 4.2** *When the delay between the spreading of $N-$fertilizer on the farm and the impact of it is increased, i.e., the higher the lag, the greater the increase in the pollution level and the higher the shadow price in the steady state, if the third-derivative $\Pi_{xx\theta}$ is negative. Otherwise, the effects are ambiguous.*

Proof is delivered in appendix D.

# 5 Discussion and perspective

To open the discussion, in this section we present a numerical application of our analytical approach. Numerical simulations are based on the following added elements, i.e. the specification of the damage function, the specification of the profit function, the specification of the density function, and a set value for parameters : The damage takes an usual quadratic form:

$$D(M) = \frac{k}{2} z M^2, k > 0 \tag{16}$$

The profit function is normalized by prices and takes a form in accordance with usual *Nitrogen*-yield functions suitable for numerous crops :

$$\Pi(x, \theta) = 1 - e^{-\theta x} - x \text{ with } \theta \in [1, e] \tag{17}$$

The function $1 - e^{-\theta x}$ refers to a yield function based on agronomic observations. We note that the third-derivative $\Pi_{xx\theta}$ is negative. Regarding the input and the output in our analysis, we consider the less performing farm such that $\theta = 1$. The best performing $\theta$ consistent with the hypothesis H4 is $e$. The contribution of farmers to a stock of pollution is considered here not $\theta-$dependent ($a(\theta) = a$ for any $\theta$). We assume that the density function follows a uniform distribution. As for the exogenous parameters, the selected values of $a$, $k$ and $\rho$ aims at clearly illustrating the different effects (noting that $\rho$ is in line with previous analytical analyses, for example the value proposed by Laffont and Tirole (1993)). The value of the discount rate, $\delta$, suits the one recommended by regulatory bodies (Lebègue et al. (2005)). According to hydro-geologists, a minimum of 30 to 60 years, depending on the aquifer, is necessary for N-fertilizer to leach into the groundwater. We set $\beta = 30$ years by default. Finally, aquifers need up to several decades to eliminate traces of N-fertilizers. We thus deduce the decline rate, $\tau = 0.04$. Table 5 resumes the values of parameters.

| a | k | $\rho$ | $\delta$ | $\beta$ | $\tau$ |
|---|---|---|---|---|---|
| 0.4 | 0.1 | 0.02 | 0.04 | 30 | 0.04 |

After optimization and solving in perfect information, we obtain the phase diagram illustrated by figure 2.

On this figure, the dashed curves represent the set of points for which time-derivatives (respectively $z$ and $\lambda$ ) are equal to 0. The two other curves (green and red) passing through the steady state describe respectively the convergent (green) and divergent (red) paths. The optimal path is the green one.

Figure 3 illustrates the lag effect, when we focus on the steady state and on the optimal path for three values of $\beta$, including the case $\beta = 0$ (i.e. no lag) and the two others lags, respectively $\beta = 15$ and $\beta = 30$ (years). Green paths starting from the initial state $z0 + \beta$ (i.e. the red point on the right) match colored points related to ten-year steps and tend to the steady state at time $\infty$ (the big blue circle on the figure). The lowest point in $z$ and $\lambda$ refers to the steady state corresponding to $\beta = 0$, the highest is the one corresponding to $\beta = 30$ years and the intermediate one is for $\beta = 15$ years. In our example the introduction of a time lag of 15 years increases the pollution stock by fifty percent in the steady state. A lag time of 30 years would double the pollution stock. Meanwhile the higher is the lag time, significantly the higher is the shadow price of the pollution.

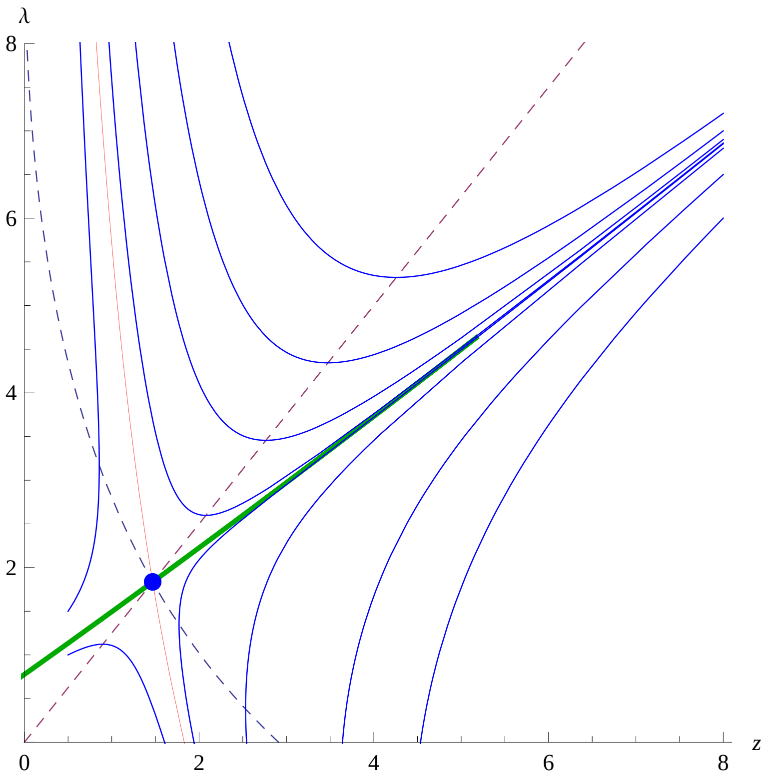


Figure 2: Numerical phase diagram in the case of perfect information describing the paths linking the pollution state $z$ and its shadow price $\lambda$.

An illustration of the lag influence on the dynamics regarding the pollution stock $z$ is displayed on figure 4. The lag obviously does not only impact the steady state. The pollution stock goes on increasing during the time interval $]0, \beta]$. In other words the time lag modifies all the dynamics.

Asymmetric information implies a cost on the regulatory body side through the informational rent paid to the farmers. The production allowed each farmer is higher than in the case of perfect information. However, some farmers can do no better than not to produce and therefore they receive a subsidy as a compensation for the income loss. The global effect on pollution stock is ambiguous (see proposition 4.2). Regarding our profit function and its negative third-derivative $\Pi_{xx\theta}$, the level of the pollution stock increases when we move from perfect information toward asymmetric information. The time lag effect is amplified in case of asymmetric information. Figure 5 illustrates both the steady state in perfect and asymmetric information for different values of the time lag and for different values of the opportunity cost of public funds.

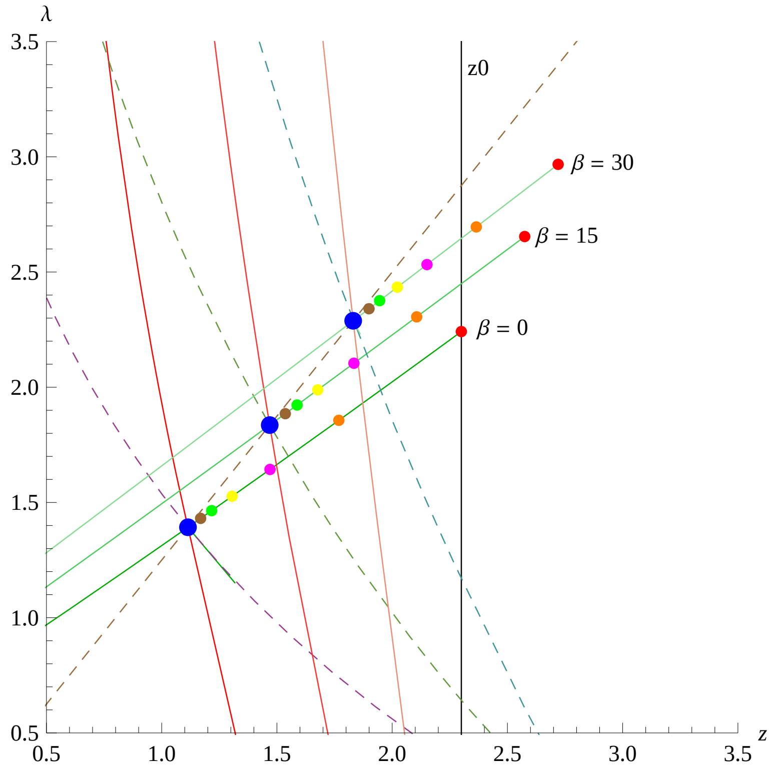


Figure 3: Comparison of both steady state and optimal dynamics regarding the pollution state $z$ and its shadow price $\lambda$ in case of perfect information, for different values of the time lag $\beta$ (respectively 0, 15, 30 years). The colored points describe the path along time (from the right at time $\beta$ to the left, by ten-year steps).

Even when asymmetric information leads to increase the stock pollution and the shadow price in the steady state, its impact appears as less important than the time lag impact.

The qualitative results provided in this paper should help modellers and decision makers to design environmental policies. Added simulations based on more realistic parameters regarding the crop system and the hydrological system should be carried out at the appropriate scale, given the European Union policy context and directives focusing on nitrate pollution and water quality.

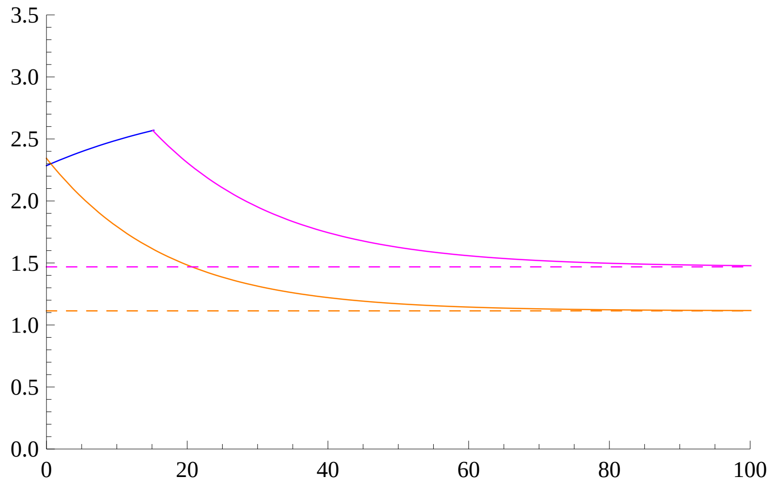


Figure 4: The dynamics of the pollution stock when $\beta = 0$ (orange curve) and when $\beta = 15$ (blue and purple curves).

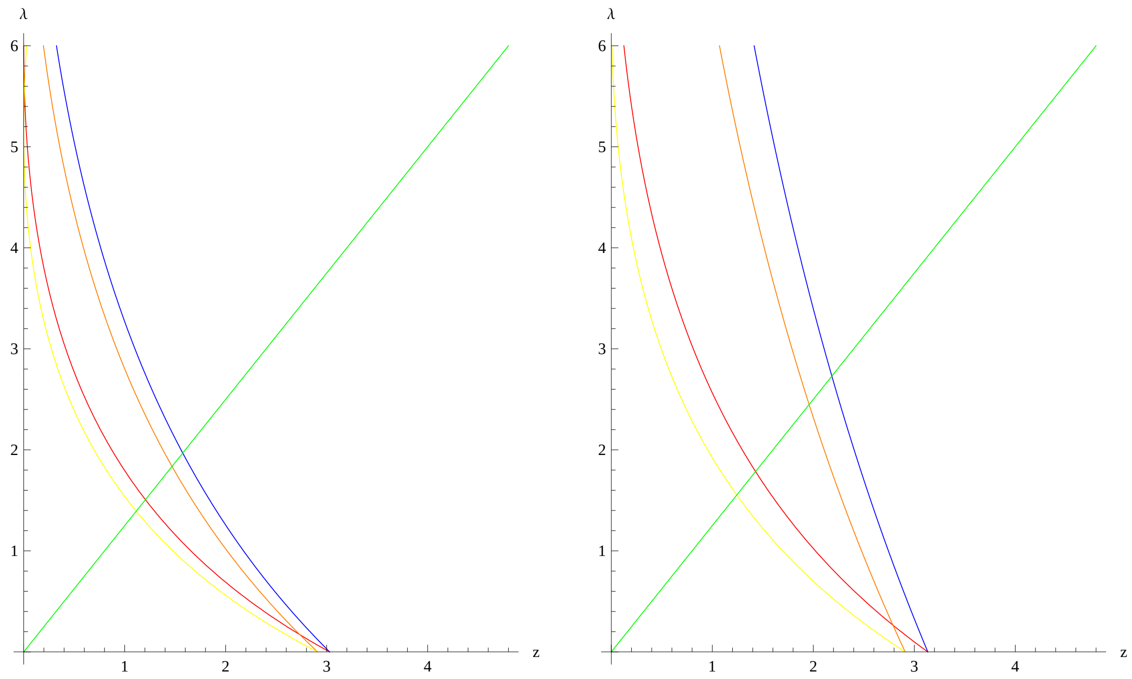


Figure 5: Impacts of the time lag $\beta$ and of the opportunity cost of public funds $\rho$ on the steady state, given perfect and asymmetric information : the steady state results of matching the green curve (i.e. the optimal path) and the yellow, purple, orange and blue curves which respectively relate to $\beta = 0$ and $\beta = 30$ in perfect information, and $\beta = 0$ and $\beta = 30$ in asymmetric information, when $\rho = 0.2$ on the left and $\rho = 0.5$ on the right.

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**APPENDIX**

# A Uniqueness of the steady state (R2)

Let us consider the set of points $\{(z(t), \lambda(t));\ \dot{z} = 0\}$. The relation $\dot{z} = 0$ is equivalently verified when $\tau z = \int_\Theta a(\theta)\phi(\theta, a(\theta)\lambda^*)$. The relation R1 implies that $z$ decreases when $\lambda$ increases. The considered set of points is a continuous curve of positive value points meeting the $z$-axis and the $\lambda$-axis.

Let us consider now the set of points $\{(z(t), \lambda(t));\ \dot{\lambda} = 0\}$, i.e. $(\delta + \tau)\lambda = D_z e^{-\delta\beta}$. The assumption (H6) immediately leads to an increasing curve starting from 0 on the $\{z, \lambda\}$ plane.

There is only one crossing point belonging to the two previous sets.

For further demonstration, any $x$ variable at the steady state is denoted by $\bar{x}$.

# B Propositions 3.1, 3.2, 3.3, 3.4

For propositions 3.1, 3.2 and 3.3 we start from the R2-system at the steady state :

$$\begin{cases} \bar{x_2}(\theta) = \phi(\theta, a(\theta)\bar{\lambda_2}e^{-\delta\beta}) \\ \tau\bar{z_2} = \displaystyle\int_\theta \bar{x_2}\gamma(\theta)d\theta \\ (\rho+\delta)\bar{\lambda_2} = D'(\bar{z_2}) \end{cases}$$

## B.1 Influence of $\beta$

We differentiate the previous system with respect to $\beta$.

$$\begin{cases} \frac{\partial \bar{x_2}}{\partial \beta} = \phi_c(-\delta e^{-\delta\beta} a(\theta)\bar{\lambda_2} + a(\theta)\frac{\partial \bar{\lambda_2}}{\partial \beta} e^{-\delta\beta}) & \text{(a1)} \\ \tau \frac{\partial \bar{z_2}}{\partial \beta} = \int_\theta a(\theta)\gamma(\theta)\frac{\partial \bar{x_2}}{\partial \beta} d\theta & \text{(a2)} \\ (\rho+\delta)\frac{\partial \bar{\lambda_2}}{\partial \beta} = D''(z_2)\frac{\partial \bar{z_2}}{\partial \beta} & \text{(a3)} \end{cases}$$

Solving (a3) for $\frac{\partial \bar{\lambda_2}}{\partial \beta}$ and substituting into (a1), we get :

$$\frac{\partial \bar{x_2}}{\partial \beta} = \phi_c e^{-\delta\beta}(-\delta a(\theta)\bar{\lambda_2} + a(\theta)\frac{D''(\bar{z_2})}{\tau+\delta}\frac{\partial \bar{z_2}}{\partial \beta}) \qquad \text{(a4)}$$

Combining (a2) and (a4), we eliminate the term $\frac{\partial \bar{x_2}}{\partial \beta}$ and thus the system can be written as a single expression which depends only on $\frac{\partial \bar{z_2}}{\partial \beta}$

$$\tau \frac{\partial \bar{z_2}}{\partial \beta} = \phi_c e^{-\delta\beta} \int_\theta a(\theta)^2(-\delta\bar{\lambda_2} + \frac{D''(\bar{z_2})}{\tau+\delta}\frac{\partial \bar{z_2}}{\partial \beta})\gamma(\theta)d\theta \qquad \text{(a5)}$$

By rearranging terms (a5) can be written as:

$$(\tau - e^{-\delta\tau}\frac{D''(\bar{z_2})}{\tau+\delta})\int_\theta a(\theta)^2\gamma(\theta)\phi_c d\theta \frac{\partial \bar{z_2}}{\beta} = -\delta\bar{\lambda_2}e^{-\delta\beta}\int_\theta a^2\gamma(\theta)\phi_c d\theta$$

Since $\phi_c(\theta, c) < 0$ (R1), $\frac{\partial z_2}{\partial \beta}$ is positive. Consequently, we deduce that:

$$\frac{\partial \bar{z_2}}{\partial \beta} > 0, \text{ therefore } \frac{\partial \bar{\lambda_2}}{\partial \beta} > 0 \text{ and therefore } \int_\theta a(\theta)\gamma(\theta)\frac{\partial \bar{x_2}}{\partial \beta}d\theta > 0$$

## B.2 Influence of $\delta$

The proof is the same as above but the R2-system at the steady state is differentiated with respect to $\delta$ :

$$\begin{cases} \frac{\partial \bar{x_2}}{\partial \delta} = \phi_c a(\theta) e^{-\delta\beta}(-\beta\bar{\lambda_2} + \frac{\partial \bar{\lambda_2}}{\partial \delta}) \\ \tau \frac{\partial \bar{z_2}}{\partial \delta} = \int_\theta a(\theta)\gamma(\theta)\frac{\bar{x_2}}{\delta} d\theta \\ -(\tau+\delta)\frac{\delta \bar{\lambda_2}}{\partial \delta} + \bar{\lambda_2} = D''(z_2)\frac{\partial \bar{z_2}}{\partial \delta} \end{cases}$$

## B.3 Influence of $\tau$

The proof is the same as above but the R2-system at the steady state is differentiated with respect to $\tau$ :

$$\begin{cases} \frac{\partial \bar{x_2}}{\partial \tau} = a(\theta) e^{-\delta\beta}\phi_c \frac{\partial \bar{\lambda_2}}{\partial \tau} \\ \bar{z_2} + \bar{\tau}\frac{\partial \bar{z_2}}{\partial \tau} = \int_\theta a(\theta)\gamma(\theta)\frac{\partial \bar{x_2}}{\partial \tau} d\theta \\ \bar{\lambda} + (\tau+\delta)\frac{[}{\partial}\bar{\lambda_2}\partial\tau = D''\frac{\partial \bar{z_2}}{\partial \tau} \end{cases}$$

## B.4 Influence of $\rho$

We start from the R3-system at the steady state :

$$\begin{cases} \bar{q_3} = \phi(\theta, \frac{a(\theta)\bar{\lambda_3}e^{-\delta\beta}}{1+\rho}) \\ \bar{z_3} = \frac{1}{\tau}\int_\theta a(\theta) q_3\bar{(}\theta)\gamma(\theta) d\theta \\ \bar{\lambda_3} = \frac{D'\bar{z_3}}{\tau+\delta} \end{cases}$$

Then, we follow the procedure used above after differentiating of the system with respect to $\rho$ :

$$\begin{cases} \frac{\partial \bar{q_3}}{\partial \rho} = \phi_c \frac{(a(\theta)e^{-\delta\beta}\frac{\partial \bar{\lambda_3}}{\partial \rho}(1+\rho) + a(\theta)\bar{\lambda_3}e^{-\delta\beta})}{(1+\rho)^2} \\ \bar{z_3} + \tau\frac{\partial \bar{z_3}}{\partial \rho} = \int_\theta a(\theta)\gamma(\theta)\frac{\partial \bar{q_3}}{\partial \rho} d\theta \\ (\tau+\delta)\frac{\partial \lambda_3}{\partial \rho} = D''(\bar{z_3})\frac{\partial \bar{z_3}}{\partial \rho} \end{cases}$$

And finally, we show that: $\frac{\partial \bar{z}}{\partial \rho} > 0$ ; $\frac{\partial \bar{\lambda}}{\partial \rho} > 0$ ; $\frac{\partial \bar{x}}{\partial \rho} > 0$ ;

# C Proposition 4.1

We apply the Taylor's theorem at the first order to the $R4$-system regarding to the steady state :

$$\begin{cases} \bar{q_4} - \bar{q_3} = \dfrac{a(\theta)(\bar{\lambda_4} - \bar{\lambda_3}) - \rho \Pi_{x\theta}(q,\theta)\frac{\Gamma(\theta)}{\gamma(\theta)} - (\bar{q_4} - \bar{q_3})\Pi_{xx\theta}(\bar{q_3},\theta)}{1+\rho} \phi_c(\theta, \dfrac{a(\theta)\bar{\lambda_3}}{1+\rho}) \\ \tau(\bar{z_4} - \bar{z_4}) = \displaystyle\int_\theta a(\theta)(\bar{q_4} - \bar{q_3})\gamma d\theta \\ \bar{\lambda_4} - \bar{\lambda_3} = \dfrac{1}{\tau+\delta}(\bar{z_4} - \bar{z_3}) \end{cases}$$

(a-)

(a7)

(a8)

We rewrite (a-) :

$$(\bar{q_4} - \bar{q_3})\frac{1+\rho+\Pi_{xx\theta}(\bar{q_3})\Pi_c(\theta, \frac{a(\theta)\bar{\lambda_3}}{1+\rho})}{1+\rho} = \frac{a(\theta)(\bar{\lambda_4} - \bar{\lambda_3}) - \rho \Pi_{x\theta}(\bar{q_3})\frac{\gamma}{\Gamma}\phi_c(\theta, \frac{a(\theta)\bar{\lambda_3}}{1+\rho})}{1+\rho}$$

(a9)

$\Rightarrow$

$$(\bar{q_4} - \bar{q_3}) = \frac{a(\theta)(\bar{\lambda_4} - \bar{\lambda_3}) - \rho \Pi_{x\theta}(\bar{q_3})\frac{\gamma(\theta)}{\Gamma(\theta)}\phi_c(\theta, \frac{a(\theta)\bar{\lambda_3}}{1+\rho})}{1+\rho+\Pi_{xx\theta}(\bar{q_3})\phi_c(\theta, \frac{a(\theta)\bar{\lambda_3}}{1+\rho})}$$

by combining (a7), (a8) and (a9), we get :

$$(\bar{\lambda_4} - \bar{\lambda_3}) = \frac{1}{\tau+\delta}\int_\theta \frac{a(\theta)(\bar{\lambda_4} - \bar{\lambda_3}) - \rho \Pi_{x\theta}(\bar{q_3})\frac{\gamma(\theta)}{\Gamma(\theta)}\phi_c(\theta, \frac{a(\theta)\bar{\lambda_3}}{1+\rho})}{1+\rho+\Pi_{xx\theta}(\bar{q_3})\phi_c(\theta, \frac{a(\theta)\bar{\lambda_3}}{1+\rho})} d\theta$$

Since $\phi_c(\theta, c) <= 0$ (R1) and $\Pi_{x\theta}(q,\theta) >= 0$ (H3), if $\Pi_{xx\theta} > 0$, then $\bar{\lambda_4} - \bar{\lambda_3} > 0$. As a result, $\bar{\lambda_4} - \bar{\lambda_3} > 0 \Rightarrow \bar{z_4} - \bar{z_4} > 0$ and the global instantaneous pollution is such that $\int_\theta a(\theta)(\bar{q_4} - \bar{q_3})\gamma(\theta)d\theta > 0$.

# D Proposition 4.2

We differentiate the $R4$-system from the steady state with respect to $\beta$ :

$$
\begin{cases}
\dfrac{\partial \bar{q_4}}{\partial \beta} = \phi_c[\dfrac{a(\theta)}{1+\rho}e^{-\delta\beta}(\dfrac{\partial \bar{\lambda_4}}{\partial \beta} - \delta\bar{\lambda_4}) - \dfrac{1}{1+\rho}\Pi_{xx\theta}\dfrac{\partial \bar{q_4}}{\partial \beta}\dfrac{\Gamma(\theta)}{\gamma(\theta)}] & \text{(a10)} \\
\dfrac{\partial \bar{z_4}}{\partial \beta} = \dfrac{1}{\tau}\displaystyle\int_\theta a(\theta)\dfrac{\partial \bar{q_4}}{\partial \beta}\gamma(\theta)d\theta & \text{(a11)} \\
(\tau+\delta)\dfrac{\partial \bar{\lambda_4}}{\partial \beta} = D''(\bar{z_4})\dfrac{\partial \bar{z_4}}{\partial \beta} & \text{(a12)}
\end{cases}
$$

which after rearranging (a10) and combining (a11) and (a12), we get :

$$
\begin{cases}
\dfrac{\partial \bar{q_4}}{\partial \beta} = \dfrac{\phi_c \frac{a(\theta)}{1+\rho}e^{\delta\beta}(\frac{\partial \bar{q_4}}{\partial \beta} - \delta\bar{\lambda_4})}{1+\frac{\rho}{1+\rho}\Pi_{xx\theta}\phi_c\frac{\gamma(\theta)}{\Gamma(\theta)}} & \text{(a13)} \\
(\tau+\delta)\dfrac{\bar{\lambda_4}}{\partial \beta} = \dfrac{D''(\bar{z})}{\tau}\displaystyle\int_\theta a(\theta)\dfrac{\partial \bar{q_4}}{\partial \beta}\gamma(\theta)d\theta & \text{(a14)}
\end{cases}
$$

Combining (a13) and (a14), we deduce :

$$
(\tau+\delta)\frac{\partial \bar{\lambda_4}}{\partial \beta} = \frac{D''(\bar{z})e^{-\delta\beta}}{\tau(1+\rho)}(\frac{\partial \bar{\lambda_4}}{\partial \beta} - \delta\bar{\lambda_4})\int_\theta \frac{a^2(\theta)\phi_c\gamma(\theta)}{1+\frac{\rho}{1+\rho}\Pi_{xx\theta}\phi_c\frac{\gamma}{\Gamma}}d\theta
$$

$$
\underbrace{(\tau+\delta) - \frac{D''(\bar{z})}{\tau}\frac{e^{-\delta\beta}}{1+\rho}\int_\theta \frac{a^2(\theta)\phi_c\gamma(\theta)}{1+\frac{\rho}{1+\rho}\Pi_{xx\theta}\phi_c\frac{\gamma(\theta)}{\Gamma(\theta)}}d\theta}_{>0\ if\ \Pi_{xx\theta}<0}\frac{\partial \bar{\lambda_4}}{\partial \beta} = \underbrace{-\delta\bar{\lambda_4}\frac{D''\bar{z}}{\tau}\frac{e^{-\delta\beta}}{1+\rho}\int_\theta \frac{a^2(\theta)\phi_c\gamma(\theta)}{1+\frac{\rho}{1+\rho}}\Pi_{xx\theta}\phi_c\frac{\gamma(\theta)}{\Gamma(\theta)}d\theta}_{>0\ if\ \Pi xx\theta<0}
$$

If $\pi_{xx\theta} < 0$, then we immediately get that $\frac{\partial \bar{\lambda_4}}{\partial \beta} > 0$, $\frac{\partial \bar{z_4}}{\partial \beta} > 0$. The total amount of instantaneous polluting input at the steady state is $\frac{\partial \bar{z_4}}{\partial \beta} = \int_\theta a(\theta)\frac{\partial \bar{q_4}}{\partial \beta}\gamma(\theta)d\theta > 0$.