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Irreversible investment and information acquisition under uncertainty

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Irreversible investment and information acquisition under uncertainty *

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Abstract

We analyse the decision of an agent to invest in new industrial activities the consequences of which on people's health and the environment are initially unknown. The agent does not have the possibility of delaying her/his investment but s/he gets the opportunity to acquire information in order to reduce her/his uncertainty. We find that the agent always invests unless the cost exceeds the direct benefit, and does acquire information with a certain degree of precision. Moreover, we show that acquiring information can encourage the agent to make a larger investment. Likewise, we identify all factors that might modify the agent's decisions. We then discuss some political instruments that could make it easier for investors to both innovate and acquire information. Finally, the impact of insurance on the investment and information acquisition decisions is also examined.

Keywords: grants, information acquisition, innovation, insurance, irreversible investment, risk, uncertainty.

JEL Classification: D21, D81, D83, G22, H81.

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Introduction

Investing in new industrial activities, such as innovations, fertilizer or pesticide manufacturing or new technologies (nano technologies), generates uncertainty about the future returns from the investment. Indeed, the agent's initial knowledge of the consequences of his investment on human health and the environment is limited. To reduce this uncertainty, the agent can do some research to gain information about those consequences. This naturally raises the question of the effect of the uncertainty reduction brought by information on the investment level: Does anticipating to gain a better understanding of the costs of the damages lead agents to make larger or smaller investments?

To address this question, we analyse the irreversible investment decision of an agent who has incomplete information about the magnitude of the investment costs. We consider that the agent has the opportunity - at a cost - to collect information and to update her/his beliefs in a Bayesian fashion. Through information acquisition, s/he will develop a better understanding of the level of danger associated with her/his investment project, and can then decide to cancel his project , and therefore limit the potential damage to people's health and the environment. This approach makes it possible to acquire information while taking precautionary measures to protect people's health and the environment.

Our paper provides a review of the literature that examines the relation between irreversible investments and information acquisition. Arrow and Kurz (1970) conducted pioneering work on irreversible investments under certainty. Their work was expanded through the introduction of uncertainty (Charles and Munro, 1985, Clark, Munro and Charles, 1985, Pindyck, 1981). There is a large literature on the role of information in irreversible investment decisions (Arrow and Fisher, 1974, Crabbe, 1987, Fisher, 1978, Freeman, 1984, Henry, 1974, Dixit and Pindyck, 1994). This literature proposes a conventional approach, called the "option value" approach, in which the investment is irreversible, i.e., it cannot be recovered in the future, and the investment decisions are made under uncertainty about future returns. An agent can postpone her/his investment so as to be able to acquire more information about the possible future consequences of her/his project. This leads one to evaluate the option value of waiting in order to get new information. An important result emerges from these studies: *A rational agent who anticipates to acquire more information in future will adopt a more flexible decision today.* In other words, the agent prefers to postpone his investment in order to acquire more information about the possible future consequences of her/his investment. We propose to analyse the irreversible investment decision, made in a context of uncertainty about future returns, by an agent who does not have the possibility of postponing her/his investment. This contrasts with the standard literature and enables us to study the effect of information acquisition on the level of investment.

Furthermore, our theory is related to the real option theory (Schwartz and Trigeorgis, 2001). We consider an agent who has the possibility, at a cost, of acquiring information, by doing some research, in order to gain some knowledge about the potential risks associated with the activity s/he is considering undertaking. Acquiring information is not an obligation, just a right. Collecting information provides flexibility to the decision maker in that it enables her/him to interrupt her/ his project before the end and so to avoid endangering the environment and people's health. Integrating the Real option approach into the theories on irreversible investment and information acquisition enables us to use endogenous information in a literature that usually deals with exogenous information.

In the upcoming sections, we argue that an agent always invests in a project unless its cost exceeds its direct benefit. However, s/he might decide to invest without conducting any research into the potential risks of her/his activities. The agent does acquire information when s/he expects to reach a certain level of precision, and thus to be able to rely on that information. Thus, from a health and environmental perspective the agent makes a non-precautionary decision when s/he decides to start a project without conducting any research to obtain information about its potential risks. We examine the factors that determine both her/his investment and information acquisition decisions. The literature on the subject suggests that prior beliefs may affect individuals' decision making. Indeed Kahneman, Slovic and Tversky (1982) highlight the existence of the "availability bias" which is people's tendency to underestimate or overestimate an event's probability according to its availability in their memory, according to their past experiences and to the weight given to that event. Furthermore, in their insurance-related studies, Kunreuther et al (1978) show that prior beliefs clearly influence the decision to subscribe to an insurance against the risk. However, other subjective factors, such as the agent's expected probability of having an accident and the precision of the information s/he has, may influence his behaviour. We show that the more confident the agent is in the success of the project, the more likely s/he is to make a bigger investment. In addition, while a higher expected information precision increases the incentive to invest and to acquire information, the agent's confidence does not have a clear impact on the information acquisition decision. Moreover, we emphasize that the cost of information acquisition also affects the decision to invest and to acquire information. Indeed, we show that the cost of information acquisition indirectly influences the agent's level of investment, but directly decreases the incentive to acquire information, and in the worse case, to start the project. We then study the effect of uncertainty reduction on the agent's investment level decision. We find that when the agent does acquire information, his level of investment may be higher than without information acquisition. Indeed, if the agent thinks that a higher investment will decrease her/his potential financial loss and that the information s/he has is precise enough to be useful -i.e., the signal leads him, if necessary, to stop his project - s/he will always invest more if s/he has acquired knowledge than if s/he has not. In this situation, collecting information, and then taking precautionary measures, does not reduce the level of innovation.

Recent health and environmental policies , such as the Precautionary Principle,[1] develop awareness of the importance of protecting people's heath and the environment, but do so without undermining innovation. We propose solutions to combine innovation and information acquisition. We first suggest alternatives to increase the expected information precision. Indeed, the agent will invest and do some research if s/he anticipates that the precision of the information s/he collects will reach a certain level. Moreover, reducing the cost of information acquisition - through public information collection or State subsidies that help pay the costs of information acquisition and provide incentives to acquire information - is an alternative way of encouraging agents to innovate without neglecting the importance of research.

Finally, we introduce the insurance market and study its impact on investment and information acquisition decisions. We find that insurance increases the level of investment in new activities. However, taking an insurance is not always an agent's optimal decision and may have a pervert effect on the information acquisition decision; indeed having an insurance might cause an agent to always continue her/his project without taking into account the possibly disastrous consequences of her/his activities. The validity of making insurance compulsory in a situation of scientific uncertainty is then debatable.

The remainder of paper is organized as follows. Section 1 introduces the model. Section 2 studies the agent's optimal decisions. Section 3 analyses the effects of different parameters - prior beliefs, probabilities of damages, expected information precision and information acquisition cost - on the agent's behaviour. Then section 4 compares the agent's investment level when s/he has acquired information and when s/he has not. Section 5 presents possible tools for combining innovation and information acquisition. Finally, section 6 examines the effect of insurance on investment and information acquisition decisions. All proofs are in the appendix.

1 The model

We consider a three period model. At period 0, an agent may invest $I \geq 0$ in a project that may cause damage to people's health and to the environment.

There are two possible states of the world, H and L associated with different probabilities of damage θ^H and θ^L, respectively. We assume that state H is more dangerous

[1]From the Rio Conference, the Precautionary Principle states that: *'In order to protect the environment, the precautionary approach shall be widely applied by States according to their capabilities. Where there are threats of serious or irreversible damage, lack of full scientific certainty shall not be used as a reason for postponing cost-effective measures to prevent environmental degradation'.*

than state L, so:

$$\theta^L < \theta^H.$$

At period 0, the prior beliefs of the agent are p_0 on state H, and $1 - p_0$ on state L. The expected probability of the damage is thus given by:

$$E(\theta) = p_0\theta^H + (1 - p_0)\theta^L.$$

The agent may decide to do some research at a given cost, $C = \bar{C}$, at period 0. In this situation, s/he will acquire information through a signal $\sigma \in \{h, l\}$ on the true state of the world at period 1. However, s/he may choose to not do any research, i.e., $C = 0$, and so will be uninformed at period 1. We consider that information precision is not observable by the agent. Then the agent's decision to acquire information is based on the level of information precision s/he expects to gain. We define it by f, and we characterize it with:

$$P(h|H) = P(l|L) = f \text{ and } P(h|L) = P(l|H) = 1 - f.$$

According to Bayes' rule, the expected probability of being in state H given signal h, and the expected probability of being in state H given signal l are, respectively:

$$P(H|h) = \frac{p_0 f}{p_0 f + (1 - p_0)(1 - f)} \text{ and } P(H|l) = \frac{p_0(1 - f)}{p_0(1 - f) + (1 - p_0)f}.$$

We assume that the agent expects an imperfectly informative signal so:

$$\frac{1}{2} < f < 1.$$

At period 1, according to signal $\sigma \in \{l, h\}$, let us define $x, x_\sigma \in \{0, 1\}$ as the agent's decision to either stop , or to continue her/his project without information acquisition and with, respectively. We assume that if the uninformed agent (informed agent) stops her/his project $x = 0$ ($x_\sigma = 0$), while $x = 1$ ($x_\sigma = 1$) if s/he continues it.

At period 2, an accident might happen. If the project has been stopped at period 1, then the returns from the project are equal to zero. On the other hand, if the project has continued until period 2, it yields a pay-off equal to $R(I)$. From this pay-off must be subtracted the cost of an accident $K(I)$ that occurs with probability θ^H or θ^L depending on the state of the world. Both depend on the initial investment I. We assume that R is an increasing and concave function, and K is an increasing and convex function, such that $R(0) = 0$ and $K(0) = 0$. This means that if the agent decides not to invest in the project, and then not to begin the project, he will get neither pay-off nor potential cost.

So the inter-temporal expected pay-offs at period 2, period 1 and period 0 may be expressed recursively. If signal σ has been perceived, $V_2^R(x_\sigma, \sigma, I)$ is the expected pay-off that the agent might earn at period 2 according to strategy x_σ and investment I.

$$V_2^R(x_\sigma, \sigma, I) = x_\sigma \left[P(H|\sigma)(R(I) - \theta^H K(I)) + (1 - P(H|\sigma))(R(I) - \theta^L K(I))\right].$$

Moreover, if there is no signal, $V_2^{NR}(x, I)$ is the expected pay-off that the agent might earn at period 2 according to strategy x and investment I.

$$V_2^{NR}(x, I) = x\left[p_0(R(I) - \theta^H K(I)) + (1 - p_0)(R(I) - \theta^L K(I))\right].$$

Since, at period 1 there is no pay-off, the inter-temporal expected pay-off at period 1 is equal to the one at period 2 thus:

$$V_1^R(x_\sigma, \sigma, I) = V_2^R(x_\sigma, \sigma, I) \text{ and } V_1^{NR}(x, I) = V_2^{NR}(x, I).$$

Finally, the inter-temporal expected pay-off for an informed agent and an uninformed agent at period 0 can be expressed as follows, respectively:

$$V_0^R(x_h, x_l, I) = -I - \bar{C} + [p_0 f + (1-p_0)(1-f)]V_2^R(x_h, h, I) + [(1-p_0)f + p_0(1-f)]V_2^R(x_l, l, I)$$

$$\text{and } V_0^{NR}(x, I) = -I + V_1^{NR}(x, I).$$

2 The optimal decision-making

At period 0, anticipating that s/he could stop or continue her/his project at period 1, the agent has three possibilities: First, s/he does not invest in the project, and then will not start the project; Second, s/he invests in the project and does research to acquire information on the potential risks of the project. So s/he innovates while taking precautionary measures ; Finally, s/he invests in the project, but does not do any research. Thus s/he begins the project but does not want to acquire information on the potentially risky consequences of her/his project. From the health and environmental point of view, s/he has a non-precautionary behaviour.

Using the backward induction method we then characterize the agent's optimal decisions

2.1 Stopping or continuing the project

Before making the decision to stop or to continue her/his project, the agent has the opportunity to collect information about the true state of the world at a cost $\bar{C}$, and to update his beliefs in a Bayesian fashion. The equilibrium strategy when the agent does not acquire information is x^*; the one when s/he receives signal $\sigma \in \{h, l\}$ is x_σ^* , and the revised expected probability of a damage is $E(\theta|\sigma) = P(H|\sigma)\theta^H + (1 - P(H|\sigma))\theta^L$.

An uninformed agent continues the project if s/he expects the pay off will be higher by continuing the project than by stopping it.

$$V_1^{NR}(0, I) < V_1^{NR}(1, I).$$

Similarly, an agent receiving signal $\sigma \in \{l, h\}$ continues the project if:

$$V_1^R(0, \sigma, I) < V_1^R(1, \sigma, I).$$

Thus, the following proposition gives conditions under which both the uninformed and the informed agent stop or continue the project.

Proposition 1 *(i) If $E(\theta) < \frac{R(I)}{K(I)}$, then the uninformed agent continues the project, i.e., $x^* = 1$; If $\frac{R(I)}{K(I)} < E(\theta)$, then s/he stops the project, i.e., $x^* = 0$; Finally, if $\frac{R(I)}{K(I)} = E(\theta)$, then s/he is indifferent between stopping and continuing the project, i.e., $x^* \in \{0, 1\}$.*
(ii) For $\sigma \in \{h, l\}$: If $E(\theta|\sigma) < \frac{R(I)}{K(I)}$, then the informed agent continues the project, i.e., $x_\sigma^* = 1$; If $\frac{R(I)}{K(I)} < E(\theta|\sigma)$, then s/he stops the project, i.e., $x_\sigma^* = 0$; Finally, if $\frac{R(I)}{K(I)} = E(\theta|\sigma)$, then s/he is indifferent between stopping and continuing her/his project, i.e., $x_\sigma^* \in \{0, 1\}$.

Part (i) of Proposition 1 implies that an uninformed agent has got two strategies: s/he always continues her/his project, or s/he always stops it. Indeed, without new information, the agent only takes into account her/his future pay-off. If it is positive, s/he continues the project, whereas if it is negative, s/he prefers to stop it in order not to waste money.

In addition, part (ii) of Proposition 1 suggests that an informed agent needs to choose between three strategies: either s/he always stops the project whatever the signal, either s/he always continues the project whatever the signal, or s/he stops the project when s/he receives signal h (being in the most dangerous state of the world), whereas when s/he gets signal l s/he continues it. In the last strategy, behaviour changes according to signals. Acquiring information provides the agent with a new strategy.

Likewise, we can easily verify that:

Lemma 1 *$E(\theta|l) \leq E(\theta) \leq E(\theta|h)$.*

According to Lemma 1, and by comparing parts (i) and (ii) of Proposition 1, we observe that information may lead the agent to stop the project, whereas without information the agent could decide to continue it, thus potentially endangering people's health and the environment. Information then serves as an instrument to protect the environment and people's health.

2.2 Project investment and information acquisition

We now focus on the agent's optimal decisions at period 0. At period 0, anticipating that at period 1, s/he either always decides to stop the project whatever the signal (strategy 1), or always decides to continue the project whatever the signal (strategy 2), or only chooses to continue the project if s/he receives signal l (strategy 3), the agent has the choice between: not participating in the project and not getting information in the future ($I = 0$) and $C = 0$); investing in the project ($I > 0$) and doing some research at a cost ($C = \bar{C} > 0$) in order to acquire information on the potential risks of the project; investing in the project ($I > 0$) without doing any research ($C = 0$).

Let us define by I_i^* the agent's optimal investment in the project, and C_i^* the agent's optimal information acquisition expenses under strategy i.

2.2.1 Strategy 1: Always stopping the project

The agent anticipates that s/he will always stop her/his project at period 1. Her/his expected pay off when s/he chooses to acquire information and when s/he chooses not to are, respectively:

$$V_0^R(0,0,I) = -I - \bar{C} \text{ and } V_0^{NR}(0,I) = -I.$$

S/he behaves in such as way as to maximize her/his expected profit. So for $I \geq 0$ s/he compares the pay-off s/he expects to obtain if s/he acquires information, with the pay off s/he expects to get if s/he does not get informed. S/he gets:

$$V_0^R(0,0,I) < V^{NR}(0,I).$$

Her/his best response is then to not acquire information. Next, s/he solves the two following problems in order to define her/his best investment response.

$$\max_{I \geq 0} V_0^R(0,0,I) \text{ and } \max_{I \geq 0} V_0^{NR}(0,I).$$

Since $V_0^R(0,0,I)$ and $V^{NR}(0,I)$ are decreasing with I, then the agent's best response is not to invest in the project. Overall, the optimal decisions are $I_1^* = 0$ and $C_1^* = 0$.

So when the agent anticipates that s/he will always stop the project in the future, s/he considers that the project is not profitable for him and does not want to waste money by undertaking the project. Moreover, the signal does not influence her/his decision; It is therefore useless for the agent to try and acquire the information. The agent then never starts the project and does not acquire information.

2.2.2 Strategy 2: To always continue the project

Now, the agent anticipates that s/he will always continue the project. Before investing s/he first checks that her/his project will be profitable. If s/he expects that the project will not be profitable, s/he does not participate in it. On the other hand, if s/he anticipates that the project will be profitable, s/he maximizes the pay-off in order to determine how much s/he is going to invest and to decide whether or not to look for information.

Her/his expected pay-off when s/he acquires information and when s/he does not are, respectively:

$$V_0^R(1,1,I) = -I - \bar{C} + R(I) - E(\theta)K(I) \text{ and } V_0^{NR}(1,I) = -I + R(I) - E(\theta)K(I).$$

The next lemma sums up the agent optimal decisions making under strategy 2.

Lemma 2 *(i) If for all $I > 0$:*

$$R(I) - E(\theta)K(I) \leq I$$

then the agent decides not to invest and not to do any research, i.e., $I_2^* = C_2^* = 0$.
(ii) If there exists $I_2 > 0$ such that:

$$I_2 < R(I_2) - E(\theta)K(I_2)$$

then the agent does not do any research, $C_2^* = 0$, but s/he invests in the project $I_2^* > 0$ which is characterized by

$$R'(I_2^*) - E(\theta)K'(I_2^*) = 1. \tag{1}$$

So when the agent anticipates that s/he will always continue her/his project, s/he only invests when s/he expects that the returns from the project will be higher than its costs. Yet, s/he prefers not to acquire information. Indeed, as s/he always anticipates to continue the project whatever the signal, getting information is useless for her/him. S/he then does not have to pay and waste money acquiring information. From a health and environmental point of view, this agent behaves in a non-precautionary manner. S/he is unwilling to take precautionary measures and is more concerned about how much profits s/he will make than about the health of people and of the environment.

2.2.3 Strategy 3: To continue or to stop the project according to the signal

From this point, the agent anticipates that at period 1 her/his decision will change according to the signal. S/he foresees that with a signal l, s/he will continue the project; and s/he will stop it if s/he receives signal h . His expected pay-off is as follows:

$$V_0^R(0,1,I) = -I - \bar{C} + p_0(1-f)(R(I) - \theta^H K(I)) + (1-p_0)f(R(I) - \theta^L K(I)).$$

S/he first verifies the profitability of her/his project. If the project is not profitable, s/he does not start the project; if it is profitable, s/he starts it. From the maximization of his expected pay-off, the agent defines her/his optimal decisions as follows.

Lemma 3 *(i) If for all $I > 0$*

$$p_0(1-f)(R(I)-\theta^H K(I)) + (1-p_0)f(R(I)-\theta^L K(I)) \leq I + \bar{C}$$

then the agent decides not to invest and not to conduct any research, i.e., $I_3^* = C_3^* = 0$.
(ii) If there exists $I_3 > 0$ such that

$$I_3 + \bar{C} < p_0(1-f)(R(I_3)-\theta^H K(I_3)) + (1-p_0)f(R(I_3)-\theta^L K(I_3))$$

then the agent does some research, $C_3^* = \bar{C}$, and s/he invests in the project $I_3^* > 0$ which is characterized by:

$$p_0(1-f)(R'(I_3^*)-\theta^H K'(I_3^*)) + (1-p_0)f(R'(I_3^*)-\theta^L K'(I_3^*)) = 1. \tag{2}$$

So s/he decides to invest in the project if her/his expected pay- off exceeds its costs (i.e. his investment in the project and the research expenses). By doing research about the potential risks of her/his activities, the agent actively chooses to protect the environment and people's health.

2.2.4 Choosing between the different strategies

Finally, we define I^* the optimal investment made by the agent according to the strategy s/he has adopted and C^* the optimal information acquisition expense according to the strategies adopted. To determine I^* and C^*, we compare the agent's expected pay-offs at period 0 of all strategies and select the levels of I and C that lead to the highest expected pay-off. The next proposition presents the results.

Proposition 2 *If*

$$I_3^* + \bar{C} < p_0(1-f)(R(I_3^*)-\theta^H K(I_3^*)) + (1-p_0)f(R(I_3^*)-\theta^L K(I_3^*)) \tag{3}$$

and

$$-I_2^* + R(I_2^*) - E(\theta)K(I_2^*) < -I_3^* - \bar{C} + (p_0(1-f)+(1-p_0)f)(R(I_3^*) - E(\theta|l)K(I_3^*)) \tag{4}$$

hold, then the agent invests $I^* = I_3^* > 0$ characterized by equation (2) and pays $C^* = \bar{C}$ to acquire information; Then if condition (4) does not hold and

$$I_2^* < R(I_2^*) - E(\theta)K(I_2^*) \tag{5}$$

holds, then the agent invests $I^* = I_2^* > 0$ characterized by equation (1) and does not acquire information $C^* = 0$; Finally, if conditions (3) and (5) do not hold, then the agent does not invest in the project $I^* = 0$, nor does he acquire information $C^* = 0$.

From conditions (3) and (4), the agent invests and looks for information if the investment and information acquisition costs are lower than the direct expected benefit and if the opportunity cost of acquiring information is positive. Indeed, the agent invests when s/he expects that her/his project will be profitable. Moreover, s/he does some research when s/he expects the returns to be higher if s/he does acquire information than if s/he does not. The expected information precision must reach a certain level to cause the agent to modify her/his behaviour according to the signal s/he receives.

However, if the cost of opportunity of acquiring information is positive (condition (4) does not hold) the agent will decide not to conduct any research. S/he will only invest without conducting any research if her/his project is profitable (condition (5) holds). S/he then prefers to not try and acquire information.

Finally, if with or without information the project is never profitable - i.e., conditions (3) and (5) do not hold - the agent never starts the project. This situation may occur if the cost of starting the project (investment and/or information acquisition expenses) exceeds the project's expected benefit. Moreover, this may also occur when the agent anticipates that s/he will always stop her/his project in the future. The prospect of a failure leads the agent to not start the project. Overall, the agent may behave in three different ways. Firstly, s/he might combine innovation and information acquisition. This behaviour promotes industrial development while ensuring the safety of people and of the environment. Secondly, the agent might innovate but refuse to do any research on the potential risks of the project. The agent does not take into account the potential damage her/his activities might cause. S/he is only interested in the possible profits. This behaviour may be dangerous for people's health and the environment. Finally, the agent might be so afraid of having to stop her/his project before the end and of the dangerous consequences of her/his activity that s/he prefers not to start it. This behaviour might hinder innovation. However, it is a safe behaviour for the rest of the players.

3 Sensitivity analysis

We have shown that the agent may behave in three different manners. It seems interesting to define the factors that affect the agent's investment and information acquisition decisions. We propose to analyse the effects of the following factors: prior beliefs, probabilities of damage, expected information precision and information acquisition cost.

3.1 Prior beliefs effects

Prior beliefs effects Prior beliefs are the agent's initial opinions on the true state of the world. More precisely, the agent believes that with a probability p_0 we will be in state H , i.e. in the state in which the probability that an accident will occur is the highest -

,and that we will be in state L with probability $1 - p_0$. Hence, a higher p_0 implies that the agent believes that the most dangerous state of the world (state H) is more likely to occur. In the following proposition, we sum up the effects of prior beliefs on the agent's investment and information acquisition decisions.

Proposition 3 *(i) I_2^* and I_3^* are decreasing with p_0.*
(ii) A lower p_0 increases the incentive to invest.
(iii) The effect of p_0 on the information acquisition decision is ambiguous.

Part (i) of Proposition 3 implies that a lower belief in the realization of the worst state of the world (state H) increases the level of investment in the project. Indeed, the more confident the agent is that the project will be successful, the bigger her/his investment will be.

Moreover, part (ii) of Proposition 3 emphasizes that a higher confidence in the success of the project may lead an agent to invest, whereas a lower confidence would cause her/him not to. Thus, this confidence may be a driving force behind innovation.

Finally, part (iii) of Proposition 3 highlights the ambiguous effect of the prior beliefs on the information acquisition decision. Indeed, a lower belief that the worst state of the world will occur may cause the agent to be so confident that the project will be profitable that s/he does not anticipate that he will ever have to withdraw from the project. In this situation, acquiring information would not be useful for him. Excessive confidence in the success of the project could lead the agent to make a dangerous decision, such as making a big investment without acquiring information about the possible consequences of his project. On the other hand, a lower belief that the worst state of the world will occur may cause the agent to only pay attention to the possible returns. S/he would then have more incentive to acquire information since doing so would reduce her/his uncertainty concerning the returns on her/his investment. In such a situation, a higher confidence in the success of the project enables her/him to combine innovation and information acquisition.

3.2 The effects of the probabilities of damage

The probabilities of damage, θ^H and θ^L, represent the probabilities of an accident occurring in states H and L, respectively. They are associated to both the dangerousness and the cost of the project. In our model, θ^H is higher than θ^L implying that state H is both more dangerous and more costly in case of accident than state L. The next proposition presents the effects of the probabilities of damage on the agent's decisions to invest and to acquire information .

Proposition 4 *(i) I_2^* and I_3^* are both decreasing with θ^H and θ^L.*
(ii) A lower θ^H and/or a lower θ^L increase the incentive to invest.
(iii) The effects of θ^H and θ^L on the information acquisition decision are ambiguous.

From parts (i) and (ii) of Proposition 4, we find that when the agent anticipates a lower probability of accident, s/he has more incentive to invest more in the project. A lower probability of accident is, for the agent, a source of confidence in the success of the project. This confidence leads the agent to invest more in the project and motivates her/him to start it.

Part (iii) of Proposition 4 shows the ambiguity of the effects of the probabilities of damage on the information acquisition decision. Just as in the case of the effects of prior beliefs, a lower accident probability may lead the agent to have so much confidence in her/his project that s/he might anticipate that s/he will never stop it and, s/he might, as a result, feel that s/he does not need to acquire information. On the other hand, the agent might be more interested in reducing her/his uncertainty about the future returns from her/his investment project if s/he anticipates that the risks are reduced. This gives her/him an incentive to acquire information in order to gain a better knowledge of the future returns from her/his investment.

3.3 The effects of Expected information precision

Information precision enables the agent to update her/his prior belief and then gain a better knowledge of the potential damages her/his project could cause and its possible returns. In our model, the agent expects his information precision is not observable by the agent. The agent's decision to acquire information is based on the level of information precision s/he expects to gain. S/he only knows that by doing research, s/he will learn about her/his project's consequences, but s/he does not know how reliable this information is.

According to Lemma 2 and Proposition 2, the level of investment and the profitability of the project without information acquisition are not affected by the expected information precision. Indeed, the agent who does not acquire information also does not anticipate updating her/his belief. Thus, her/his future pay-off does not depend on the expected information precision. So the agent makes his decision to invest in the project without taking this precision into account.

The next proposition describes the effects of the expected information precision on the investment and information acquisition decisions.

Proposition 5 *(i) If $R'(I_3^*) - \theta^H K'(I_3^*) \leq 0$, then I_3^* is increasing with f. On the other hand, if $R'(I_3^*) - \theta^H K'(I_3^*) > 0$ then:*

- *if $p_0 < \frac{R'(I_3^*) - \theta^L K'(I_3^*)}{(R'(I_3^*) - \theta^H K'(I_3^*)) + (R'(I_3^*) - \theta^L K'(I_3^*))}$ then I_3^* is increasing with f;*
- *otherwise, I_3^* is decreasing with f.*

(ii) If $R(I_3^*) - \theta^H K(I_3^*) \leq 0$ then a higher expected information precision increases the incentives to invest and to acquire information. Otherwise, if $R(I_3^*) - \theta^H K(I_3^*) > 0$, the expected information precision does not affect his investment decision because the agent never acquires information and always continues her/his project.

From part (i) of proposition 5, we assume that the more reliable the information, the larger the investment. However, this assumption does not hold when the agent believes that the worst state of the world is more likely to occur. In this case, if the reliability of the information increases, her/his investment will be smaller. Actually, this increases her/his fear that the project might fail.

Moreover, part (ii) of Proposition 5 implies that if the agent is afraid of suffering a loss if state H occurs, the hope of obtaining more precise information will lead her/him to invest and to do some research. Indeed, s/he may consider that her/his future knowledge might not be sufficient to make her/him stop her/his project. In addition, if the agent expects a positive pay-off in state H, s/he is not interested in acquiring information because s/he always prefers to continue the project. So in this case, the expected information precision does not affect the investment decision.

3.4 The effects of the costs of information acquisition

Private experts, private laboratories or any other private party able to provide scientific information about the dangerousness or more generally the characteristics of the agent's activities are considered the source of information in our model. The information obtained by the agent is therefore not public. The agent is willing to pay a certain amount of money to acquire this information. According to Proposition 2, the information acquisition cost $\bar{C}$ has an impact on conditions (3) and (4). If both conditions hold, and then if $\bar{C}$ increases, the conditions might no longer hold. Indeed, an increase in the cost of information acquisition might cause the project of the agent who initially wanted to acquire information, to become less profitable. In this case, the agent may decide either to invest without acquiring any information, or to not participate in the project. Thus, an increase in the cost of information acquisition might discourage the agent from acquiring information about the potential risks of the project, or in the worst case, from investing in it. In addition, since I_2^* is not necessarily equal to I_3^*, the information acquisition cost then indirectly affects the size of the investment.

4 Level of investment

In this section, we address the question of whether reducing uncertainty (i.e., acquiring information about the potential risks of damages) causes the agent to make a smaller or a bigger investment in the project. We propose to study the cases in which the project is profitable with and without information acquisition, i.e., conditions (3) and (5) hold.[2] The agent's optimal investment decisions are then either I_2^* or I_3^*. The following proposition summarizes the conditions under which I_2^* is lower than, equal to, or higher than I_3^*, whatever the agent's optimal investment decision I^*.

Proposition 6 *For* $I^* \in \{I_2^*, I_3^*\}$
(i) If $R'(I^*) - \theta^H K'(I^*) \geq 0$ *then the level of investment with information acquisition is lower than the one without, i.e.,* $I_3^* < I_2^*$.
(ii) If $R'(I^*) - \theta^H K'(I^*) < 0$ *then:*

- *if* $\frac{(1-p_0)(R'(I^*)-\theta^L K'(I^*))}{(1-p_0)(R'(I^*)-\theta^L K'(I^*))-p_0(R'(I^*)-\theta^H K'(I^*))} < f$, *the level of investment with information acquisition is higher than the one without, i.e.,* $I_2^* < I_3^*$;
- *Otherwise the level of investment with information acquisition is lower than, or equal to the one without, i.e.,* $I_3^* \leq I_2^*$.

For all $I > 0$, the direct expected benefit without information acquisition and the one with are, respectively:

$$R(I) - E(\theta) K(I) \text{ and } p_0(1-f)(R(I) - \theta^H K(I)) + (1-p_0) f(R(I) - \theta^L K(I).$$

So the marginal direct expected benefit without information acquisition and the one with are, respectively:

$$R'(I) - E(\theta) K'(I) \text{ and } p_0(1-f)(R'(I) - \theta^H K'(I)) + (1-p_0) f(R'(I) - \theta^L K'(I).$$

When we compare both marginal direct expected benefits, we note that for the same investment without and with information acquisition, $I > 0$ if

$$0 < p_0 f(R'(I) - \theta^H K'(I)) + (1-p_0)(1-f)(R'(I) - \theta^L K'(I)) \tag{6}$$

[2]Obviously, if the project is not profitable with information acquisition (i.e., condition (3) does not hold) while it is without (i.e., condition (5) holds), $I_3^* = 0 < I_2^*$. On the other hand, if the project is profitable with information acquisition (i.e., condition (3) holds) while it is not without (i.e., condition (5) does not hold), $I_2^* = 0 < I_3^*$. Finally, if the project is never profitable in both kinds of situations (i.e., conditions (3) and (5) do not hold), $I_2^* = I_3^* = 0$.

then the expected returns from I are higher without information acquisition than with.

So according to part (i) of Proposition 6 if the agent thinks that whatever her/his investment optimal decision, the expected pay-off when state H occurs is increasing with the investment (for $I^* \in \{I_2^*, I_3^*\}$ $R'(I^*) - \theta^H K'(I^*) \geq 0$, then equation (6) holds.[3] Actually, an agent who does acquire information anticipates that s/he will only continue her/his project if state L occurs. Thus the expected pay-off if state H occurs is seen as a cost. On the other hand, an agent who does not want to acquire information expects that s/he always will continue her/his project, and so s/he associates the expected pay-off if state H occurs to a potential gain. So considering that a higher investment increases the expected pay-off if state Hoccurs, the expected direct marginal benefit when the agent chooses to acquire information becomes lower than when s/he chooses not to look for information. If the optimal decision is to acquire information, and then to invest I_3^*, the agent then expects a higher return without information acquisition than with. A larger investment will be more profitable when the agent has not done any research than if s/he has. However the profit evaluated at period 0, taking into account the costs of investments and research, will be higher if the agent does some research than if s/he does not. In addition, if the optimal decision is to not acquire information, the agent invests I_2^*. As s/he expects to earn better returns without information acquisition, making a larger investment without conducting any research will be more profitable for the agent than making the investment without looking for information.

Let us now examine part (ii) of Proposition 6. If the agent thinks that whatever her/his investment optimal decision, the expected pay-offs if state H occurs is decreasing with the investment, then equation (6) does not always hold. Actually, the marginal direct expected benefit with information acquisition may be higher than, equal to, or lower than the one without. Indeed, with a higher investment, an agent who does acquire information decreases her/his potential loss if state H happens and increases his expected gain if state L occurs. Moreover, since $R'(I) - \theta^H K'(I) < R'(I) - \theta^L K'(I)$ a higher investment, in the case of an agent who does not acquire information, leads to a decrease in her/his expected gain if state H occurs, whereas her/his expected gain if state L happens increases. It is then not straightforward to conclude on the ranking between the expected direct marginal benefits with information acquisition and those without. The expected information precision has to reach a certain level for the agent to be confident about the project consequences and for her/him to make a higher investment with information acquisition than without.

Overall, when the project is profitable, the agent invests but her/his behaviour is more or less dangerous for the health of people and of the environment. Indeed, the agent may decide to make a higher investment without taking precautionary measures. This

[3] According to Lemmas 2 and 3, for $I^* \in \{I_2^*, I_3^*\}$ we always get $0 < R'(I^*) - \theta^L K'(I^*)$.

behaviour puts the safety of the human beings and of the environment in huge danger. For example, producing a large quantity of new medicines without doing any research on the risks for human health associated with these medicines, seems irresponsible. On the other hand, the agent may decide to make a smaller investment than the one s/he could undertake with information acquisition. In this case, the innovation is slowed down but this does not mean that people's safety is guaranteed. In a situation of uncertainty, it is wise to acquire information. Nevertheless, this may lead the agent to reduce her/his investment. Indeed, in the few past decades, the cost of research in the chemical industry has led entrepreneurs to reduce their investments in new medicines. But, although the size of investments has decreased, the number of innovations has not.

5 Combining innovation and information acquisition

Innovation is necessary for industrial and economic development. Yet, when they make their investment decisions, agents have limited initial knowledge about the risks of their projects for people's health and for the environment. In order to reduce the uncertainty characterizing innovation, precautionary measures -such as acquiring better knowledge about the potential damage the new activities might cause - can be implemented. Besides, the European Directives have placed emphasis on the necessity to find a balance between precautionary measures and innovation.

We propose some solutions in order to combine innovation and information acquisition. As mentioned in the previous sections, an increase in the expected information precision and a decrease in the cost of information acquisition would enable potential investors to combine information acquisition and innovation. Indeed, a higher confidence in the accuracy of the results of the research, makes information more reliable for the agent. Moreover, a reduction in the cost of information acquisition reduces the total cost of the project, which implies that the latter might be more profitable when the agent has acquired information than when s/he has not.

However, independently of the expected information precision and the information acquisition cost, the agent may want to invest without taking precautionary measures. What solutions would then encourage agents to invest in innovation projects and do some research at the same time?

5.1 Increasing the expected information precision

Before making her/his decision to invest and to acquire information, the agent expects a certain level of information precision. S/he then anticipates the quality of the results obtained by the research team. A higher confidence in the quality of her/his research team increases her/his expectation of obtaining reliable information. This confidence comes from the agent's evaluation of the efficiency of the research team. This efficiency depends, first of all, on the team's integrity and effort. Indeed, an agent has more confidence in her/his own research team, or in an independent one, than in a team that is financed by a lobby. Indeed, lobbies, such as pharmaceutical or chemical lobbies, may provide forged results in order to promote their cause.

Moreover the reputation and publications of the researchers are good indicators of the ability of a research team to produce accurate results. However, hiring an efficient research team is no easy task. Indeed, as Laffont and Martimort (2002) show in their principal-agent model, the employer can face "adverse selection problems" which make it difficult to evaluate the real abilities of employees.

In addition, an agent could hire different research teams. By comparing their results, s/he could gain a better understanding of the potential risks of her/his new activity and reduce the effects of the adverse selection problem.

5.2 Reducing the cost of information acquisition

Public research centres (BARPI and al, 2005), or observation of the adoption of new products or discussions with other agents (McCardle, 1985, Wozniak, 1993, Sinclair Desgagné and Gozlan, 2003) are sources of free information for the agent. However, as the innovative activities involve new risks, free information may not answer the agent's concerns. Public, free information may not be a substitute for private and therefore costly information.

The State could pay, through subsidies, part of or all the costs of information acquisition incurred by the agent. This type of intervention would be useful in situations where an agent wants to start her/his project but cannot take precautionary measures because of the cost of information acquisition.[4]

Formally, the State could give the agent a subsidy. This subsidy would be directly used for conducting research. The agent would have to be indifferent between investing and acquiring information with a subsidy, and investing without information. So S_1 would be defined such that:

$$V_0^R(0,1,I_3^*) + S_1 = V_0^{NR}(1,I_2^*).$$

So thanks to the intervention of the State , the agent would be willing to invest and take precautionary measures. However, her/his investment in the project might then be lower than it would have been if s/he had undertaken the project without acquiring information. In this situation, an additional grant might help increase the investment level. Nevertheless, this intervention could reduce the general welfare. Indeed, the agent

[4]Obviously, if it is not optimal to start the project, State has not to make an intervention. The project is not profitable.

would earn lower returns on his investments, and taxpayers would have to finance this subsidy. The right balance between promoting a high level of innovation and protecting the social welfare will have to be found.

5.3 When the agent chooses to not look for information

An agent may prefer not to acquire information because his expected pay-off - even if information is free and perfect - is higher than the pay-off with information acquisition. This situation is very dangerous for society. The agent does not take the precautionary measures that would protect the safety of people and of the environment. The State could then intervene in two ways: give the agent a subsidy, or implement regulations.

We first analyse the situation in which the State gives the agent a subsidy. Formally, the State could give a subsidy $0 < S_2$ to the agent in order to compensate her/him for the costs incurred in acquiring information. The agent would have to be indifferent between investing and acquiring information thanks to a State subsidy and investing without acquiring any information. So S_2 would be defined such that:

$$V_0^R(0, 1, I_3^*) + S_2 = V_0^{NR}(1, I_2^*).$$

With this subsidy, the agent would innovate and take precautionary measures. However, this measure would be costly for society. A system of risk prevention regulations could be implemented in order to encourage investors to take risks into account. In our model, the agent is already constrained by the strict liability rule, i.e., if an accident occurs the agent is liable for damages and must pay for them (Shavell, 1980 and Miceli, 1997). This rule complies with the polluter-payer principle established in Europe. A committee could also evaluate the profitability of the project by taking into account the cost $\bar{C}$ of conducting research. If the project is profitable, the Court of law could, by using the threat of financial punishment $T > 0$, force the agent to conduct some research on the potential risks of his project. The agent would have to be indifferent between investing and acquiring information, and investing without acquiring information and paying the transfer T. So T would be defined such that:

$$V_0^R(0, 1, I_3^*) = V_0^{NR}(1, I_2^*) - T.$$

Through this sanction, the State forces the agent to be aware of the importance of acquiring information on the potential risks of his project.

6 Insurance

Insurance is a risk sharing tool that is widely used in the industrial sector. However, insurance companies are cautious about insuring firms against this kind of risks. They do not want their role to be restricted to risk coverage, but also wish to be a driving force encouraging agents to learn about the potential risks of their industrial activities and take them into account when investing in innovation projects.

In this section, we examine the effect of insurance on the agent's decisions to invest and to acquire information. We consider that an insurer offers to help an agent to live up to her/his responsibility by covering part of the costs of potential damages. The agent has to define his insurance demand; s/he then evaluates whether the optimal strategy is to get insured and what level of insurance cover s/he needs.

We analyse the same model as in the previous sections, but we extend it by considering that in exchange for a premium $P > 0$ the insurer agrees to pay part of the costs of damages that might be caused by the agent in the case of an accident. Now, the cost, K, depends both on the investment in the project and the level of the premium, i.e., $K(I, P)$. Without any insurance, the cost is equal to the one calculated in the previous sections, i.e., for all $I \geq 0$ we get $K(I, 0) = K(I)$, and without investment, there is no cost, i.e., for all $P > 0$ we get $K(0, P) = 0$.

We assume that K is increasing and convex with I and it is decreasing and convex with P. Moreover, we suppose that for all $I > 0$ and $P > 0$:

$$\frac{\partial^2 K(I,P)}{\partial I^2}\frac{\partial^2 K(I,P)}{\partial P^2} - \left[\frac{\partial^2 K(I,P)}{\partial I \partial P}\right]^2 > 0.$$

Thus, we consider that there exist an investment and a premium that enable the agent to minimize the cost of damages in case of an accident. The agent follows the same timing as previously, except that her/his set of possible decisions at period 0 has increased. Indeed, at period 0, the agent has to choose between: not participating in the project, not acquiring information and not getting an insurance ($I = 0$, $C = 0$ and $P = 0$); investing in the project ($I > 0$), doing some research at a cost ($C = \bar{C}$), and getting an insurance ($P > 0$); investing in the project ($I > 0$), doing some research at a cost ($C = \bar{C}$), and not getting insured ($P = 0$); investing in the project ($I > 0$) without doing any research ($C = 0$), but getting an insurance ($P > 0$); investing in the project ($I > 0$) without doing any research ($C = 0$) and without getting an insurance ($P = 0$).

The inter-temporal expected pay-offs with insurance at period 2, period 1 and period 0 may be expressed, recursively. Since, at period 1 there is no pay-off, the inter- temporal expected pay-off at period 1 is equal to the one at period 2, thus: If signal $\sigma \in \{h, l\}$ has been perceived, the expected pay-off the agent might earn at period 2 and at period 1 according to strategy x_σ, investment I and premium P is:

$$V_2^{RI}(x_\sigma, \sigma, I, P) = V_1^{RI}(x_\sigma, \sigma, I, P) = x_\sigma[P(H|\sigma)(R(I) - \theta^H K(I,P)) + (1 - P(H|\sigma))(R(I) - \theta^L K(I,P))].$$

If there is no signal, $V_2^{NRI}(x, I, P)$ is the expected pay-off that the agent might earn at period 2 and at period 1 according to strategy x, investment I and premium P.

$$V_2^{NRI}(x,I,P) = V_1^{NRI}(x,I,P) = x[p_0(R(I)-\theta^H K(I,P)) + (1-p_0)(R(I)-\theta^L K(I,P))].$$

Finally, at period 0, the agent's inter-temporal expected pay-off with an insurance and information acquisition and that with an insurance but without information acquisition can be expressed as follows, respectively,

$$\begin{aligned} V_0^{RI}(x_h,x_l,I,P) = \quad & -I-P-\bar{C}+[p_0 f+(1-p_0)(1-f)]V_2^{RI}(x_h,h,I,P) \\ & +[(1-p_0)f+p_0(1-f)]V_2^{RI}(x_l,l,I,P) \end{aligned}$$

$$\text{and } V_0^{NRI}(x,I,P) = -I-P+V_1^{NRI}(x,I,P).$$

6.1 Stopping and continuing the project

Let us, now discuss the conditions in which an agent interrupts her/his project and those in which s/he continues with the project. The agent has the possibility to get an insurance. We define the equilibrium strategy when the agent does not acquire information by x^{**}, and for $\sigma \in \{h,l\}$ the equilibrium strategy by x_σ^{**}. Thus, given I and P the conditions in which the agent withdraws from the project and those in which s/he continues the project are:

Proposition 7 *(i) If $E(\theta) < \frac{R(I)}{K(I,P)}$, then the agent continues the project, i.e., $x^{**}=1$; If $\frac{R(I)}{K(I,P)} < E(\theta)$, then the agent withdraws from the project, i.e., $x^{**}=0$; Finally, if $\frac{R(I)}{K(I,P)} = E(\theta)$, then the agent is indifferent between stopping and continuing her/his project, i.e., $x^{**} \in \{0,1\}$.*
(ii) For $\sigma \in \{h,l\}$: If $E(\theta|\sigma) < \frac{R(I)}{K(I,P)}$, then the agent continues the project, i.e., $x_\sigma^{**}=1$; If $\frac{R(I)}{K(I,P)} < E(\theta|\sigma)$, then the agent stops the project, i.e., $x_\sigma^{**}=0$; Finally, if $\frac{R(I)}{K(I,P)} = E(\theta|\sigma)$, then the agent is indifferent between stopping and continuing her/his project, i.e., $x_\sigma^{**} \in \{0,1\}$.

The conditions in which the agents interrupts and continues the project only differ from the previously described conditions in that term $K(I)$ now becomes $K(I,P)$. So we get the same possible strategies as in the previous sections. However, according to Proposition 1 and since the cost with an insurance is lower than, or equal to the cost without an insurance, i.e., for all $I \geq 0$ and $P > 0$ we get $K(I,P) \leq K(I)$, an insurance gives the agent more incentive to continue her/his project.

6.2 Project investment and information acquisition

We now present the agent's optimal decisions at period 0. As previously, the agent makes her/his decisions anticipating that at period 1 s/he will either decide to always stop the project (strategy 1), or to always continue it (strategy 2), or to only continue it if s/he receives signal l (strategy 3).

Let us define by I_i^{**} the agent's optimal investment in the project, C_i^{**} the agent's optimal information acquisition expense, and P_i^{**} the agent's optimal premium under strategy i.

6.2.1 Strategy 1: Always stop the project

The agent anticipates that s/he will always stop her/his project. The expected pay-off with information acquisition and the pay-off without, are, respectively:

$$V_0^{RI}(0,0,I,P) = -I - \bar{C} - P \text{ and } V_0^{NRI}(0,I,P) = -I - P.$$

S/he behaves in such a way as to maximize her/his expected profit. So for $I \geq 0$ and $P \geq 0$ given, s/he compares the expected pay-off when s/he acquires information with the expected pay-off when s/he does not. S/he gets:

$$V_0^{RI}(0,0,I,P) < V^{NRI}(0,I,P).$$

His best response is then not to acquire information. Next, s/he solves the two following problems in order to make her/his best investment and insurance premium choices.

$$\max_{I,P\geq 0} V_0^{RI}(0,0,I,P) \text{ and } \max_{I,P\geq 0} V_0^{NRI}(0,I,P).$$

Since $V_0^{RI}(0,0,I,P)$ and $V^{NRI}(0,I,P)$ are decreasing with both I and P , then the agent's best response is not to invest in the project and not to subscribe to any insurance plan. So $I_1^{**} = 0$, $C_1^{**} = 0$ and $P_1^{**} = 0$.

Overall, when the agent anticipates that s/he will always stop her/his project, s/he prefers not to start it, and it is then not useful for him to acquire information and to subscribe to an insurance. This would be a waste of money.

6.2.2 Strategy 2: To always continue the project

Now, the agent anticipates that s/he will always continue the project. The expected pay-off from her/his investment, with information acquisition, and the pay-off without, are, respectively:

$$V_0^{RI}(1,1,I,P) = -I - \bar{C} - P + R(I) - E(\theta)K(I,P) \text{ and } V_0^{NRI}(1,I,P) = -I - P + R(I) - E(\theta)K(I,P).$$

In order to make the best decision about her/his investment, information acquisition and insurance premium, the agent maximizes his expected profit. So for $I \geq 0$ and $P \geq 0$, s/he compares the expected pay-off from her/his investment when s/he acquires information and the pay-off when s/he does not. S/he gets:

$$V_0^{RI}(1,1,I,P) < V^{NRI}(1,I,P).$$

His best response is then not to acquire information, $C_2^{**} = 0$. When the project is profitable, we now define by I_2^{BR} the agent's best response for the investment in the project, and P_2^{BR} the agent's best response for the premium.

In order to find I_2^{BR} and P_2^{BR}, the agent solves the following two problems:

$$\max_{I,P\geq 0} V_0^{RI}(1,1,I,P) \text{ and } \max_{I,P\geq 0} V_0^{NRI}(1,I,P).$$

The solutions are summed up in the following lemma.

Lemma 4 *I_2^{BR} and P_2^{BR} are characterized by:*

$$R'(I_2^{BR}) - E(\theta)\left[\frac{\partial K(I_2^{BR}, P_2^{BR})}{\partial I} - \frac{\partial K(I_2^{BR}, P_2^{BR})}{\partial P}\right] = 0. \tag{7}$$

So, the agent's optimal decisions are given in the following lemma.

Lemma 5 *(i) If for all $I > 0$ and $P \geq 0$ we have $R(I) - E(\theta)K(I,P) \leq I + P$ then the agent does not invest, nor does s/he do any research and or get an insurance, i.e., $I_2^{**} = C_2^{**} = P_2^{**} = 0$.*
(ii) If there exists $I_2 > 0$ and $P_2 \geq 0$ such that $I_2 + P_2 < R(I_2) - E(\theta)K(I_2, P_2)$ then if

$$E(\theta)\left(K(I_2^{BR}) - K(I_2^{BR}, P_2^{BR})\right) \leq P_2^{BR},$$

the agent invests in the project $I_2^{**} = I_2^* > 0$ characterized by equation (1), but s/he does not conduct any research and does not get an insurance, i.e., $C_2^{**} = 0$ and $P_2^{**} = 0$; Otherwise, the agent invests in the project $I_2^{**} = I_2^{BR} > 0$ and subscribes to an insurance, $P_2^{**} = P_2^{BR} > 0$ characterized by equation (7), but s/he does not conduct any research, $C_2^{**} = 0$.

The agent always invests in the project when its costs (investment and premium) are lower than its expected returns. Her/his subscription to an insurance therefore depends on the premium s/he would need to pay. Indeed, if there is an opportunity cost of getting an insurance, the agent does not get an insurance. Moreover, s/he never acquires information because s/he anticipates that s/he will continue the project whatever signal s/he receives. Then the signal is not useful to her/him. S/he prefers not to waste money conducting research.

6.2.3 Strategy 3: continuing or stopping the project according to the signal

The agent now anticipates that with a signal l, s/he will continue the project, whereas if s/he receives a signal h s/he will stop the project. S/he foresees that s/he will acquire information at period 1, and then is willing to pay an expense $\bar{C}$ at period 0. So s/he only considers these two following expected pay-offs with and without insurance respectively:

$$V_0^{NRI}(0,1,I,\bar{C},P) = -I-\bar{C}-P+p_0(1-f)(R(I)-\theta^H K(I,P))+(1-p_0)f(R(I)-\theta^L K(I,P))$$

and

$$V_0^{NR}(0,1,I,\bar{C}) = -I-\bar{C}+p_0(1-f)(R(I)-\theta^H K(I))+(1-p_0)f(R(I)-\theta^L K(I))$$

When the project is profitable, we define by I_3^{BR} the agent's best investment choice and P_3^{BR} the agent's best insurance premium choice. The agent then solves the following problem.

$$\max_{I,P\geq 0} V_0^{RI}(0,1,I,P).$$

The next lemma characterizes this problem's solutions.

Lemma 6 *I_3^{BR} and P_3^{BR} are such that:*

$$(p_0(1-f)+(1-p_0)f)R'(I_3^{BR})-(p_0(1-f)\theta^H+(1-p_0)f\theta^L)\left[\frac{\partial K(I_3^{BR},P_3^{BR})}{\partial I}-\frac{\partial K(I_3^{BR},P_3^{BR})}{\partial P}\right]=0. \quad (8)$$

So, the optimal decisions are then defined as follows.

Lemma 7 *(i) If for all $I>0$ and $P\geq 0$*

$$p_0(1-f)(R(I)-\theta^H K(I,P))+(1-p_0)f(R(I)-\theta^L K(I,P))\leq I+\bar{C}+P$$

then the agent does not invest, neither does s/he do any research or get an insurance, i.e., $I_3^{**}=C_3^{**}=P_3^{**}=0$.
(ii) If there exists $I_3>0$ and $P_3\geq 0$ such that

$$I_3+P_3+\bar{C}<p_0(1-f)(R(I_3)-\theta^H K(I_3,P_3))+(1-p_0)f(R(I_3)-\theta^L K(I_3,P_3))$$

then if

$$(p_0(1-f)\theta^H+(1-p_0)f\theta^L)\left(K(I_3^{BR})-K(I_3^{BR},P_3^{BR})\right)\leq P_3^{BR}$$

then the agent invests in the project $I_3^{**}=I_3^*>0$ characterized by equation (2), does some research $C_3^{**}=\bar{C}$ but does not get insured $P_3^{**}=0$; Otherwise, the agent invests in the project $I_3^{**}=I_3^{BR}>0$, subscribes to an insurance, $P_3^{**}=P_3^{BR}$ characterized by equation (8), and conducts some research $C_3^{**}=\bar{C}$.

So the agent invests and subscribes to an insurance if the expected profit from the project exceeds its costs (investment, information acquisition cost and premium) and if there is an opportunity cost of not getting insured. Moreover, the agent wants to acquire information because s/he anticipates that the signal will cause her/him to make different decisions.

6.2.4 Choosing between the different strategies

Finally, we define I^{**} as the optimal investment in the project over all the strategies, C^{**} as the optimal cost of acquiring information over all the strategies, and P^{**} the optimal premium over all the strategies. To determine I^{**}, C^{**} and P^{**}, we compare the agent's expected pay-offs at period 0 for all strategies and select the levels of I, C and P that lead to the highest expected pay-off. The next proposition sums up all the decisions.

Proposition 8 *(i) If*

$$I_3^{**}+\bar{C}+P_3^{**} < p_0(1-f)(R(I_3^{**})-\theta^H K(I_3^{**},P_3^{**}))+(1-p_0)f(R(I_3^{**})-\theta^L K(I_3^{**},P_3^{**})), \quad (9)$$

$$V_0^{NRI}(1,1,I_2^{**},P_2^{**}) < V_0^{RI}(0,1,I_3^{**},P_3^{**}) \quad (10)$$

$$\text{and } P_3^{**} < \left(p_0(1-f)\theta^H + (1-p_0)f\theta^L\right)\left(K(I_3^{**}) - K(I_3^{**},P_3^{**})\right) \quad (11)$$

hold, then the agent invests $I^{**} = I_3^{BR} > 0$, chooses a premium $P^{**} = P_3^{BR} > 0$ characterized by equation (8) and pays $C^{**} = \bar{C}$ to acquire information. However, if condition (11) does not hold while conditions (9) and (10) hold, the agent invests $I^{**} = I_3^* > 0$ characterized by equation (2), pays $C^{**} = \bar{C}$ to acquire information but does not get an insurance $P^{**} = 0$.
(ii) If

$$I_2^{**} + P_2^{**} < R\left(I_2^{**}\right) - E\left(\theta\right) K\left(I_2^{**}, P_2^{**}\right) \quad (12)$$

$$\text{and } P_2^{**} < E(\theta)\left(K(I_2^{**}) - K(I_2^{**},P_2^{**})\right) \quad (13)$$

hold and condition (10) does not hold, then the agent invests $I^{**} = I_2^{BR} > 0$ and chooses a premium $P^{**} = P_2^{BR} > 0$ characterized by equation (7) but does not acquire information $C^{**} = 0$; otherwise if conditions (10) and (13) do not hold while condition (12) holds, then the agent invests $I^{**} = I_2^* > 0$ characterized by equation (1), but s/he does not acquire information $C^{**} = 0$, nor does s/he get an insurance $P^{**} = 0$.
(iii) If conditions (9) and (12) do not hold then the agent does not invest in the project, $I^{**} = 0$, nor does s/he acquires information $C^{**} = 0$ or get an insurance $P^{**} = 0$.

First, Proposition 8 implies that the agent's investment with and without an insurance may be different. This raises the question of the effect of the insurance on the investment level. The following lemma answers this question.

Lemma 8 *When the agent decides to get an insurance, whatever her/his decision concerning information acquisition, s/he makes a larger investment than s/he would without an insurance.*

So being insured seems to be conducive to a higher level of investment in the new activities. Getting an insurance reduces the agent's fear of losing money in the event of an accident and then increases his level of investment.

Then, according to Proposition 8, the agent decides to invest in the project when it is profitable, i.e., when condition (9) and/or condition (12) hold. Moreover, s/he chooses to acquire information when her/his project is more profitable if s/he does acquire information and the opportunity cost of acquiring it is positive (conditions (9) and (10) hold). In addition, her/his decision concerning insurance depends on the premium s/he would have to pay. If the opportunity cost of insurance is negative (conditions (11) and (13) do not hold), the agent never subscribes to an insurance.

So getting insured is not always an optimal decision for the agent. If insurance becomes compulsory, some innovation projects will not start. Indeed, the reduction of the costs of damages may not compensate for the cost of the insurance premium. This may reduce the incentives to innovate. Moreover, insurance may have a pervert effect on the information acquisition decision. Indeed, according to Proposition 7, insurance increases the incentive to continue the project at period 1. As a result, the agent may lose interest in acquiring information. Thus, insurance may discourage, rather than encourage, agents to combine innovation and information acquisition.

Furthermore, we emphasize that from the insurer's point of view, the premium definition may be complicated. Indeed, the insurer does not have enough knowledge about the agent's investment and benefit and the potential costs of an accident. This creates a moral hazard and adverse selection problems for the insurer. These types of problems may reduce both information acquisition and innovation. In addition, the insurer may have little incentive to insure new activities. Indeed, innovation projects may create huge financial damages. Schemes, such as mutual insurance or re-insurance systems can, however, reassure insurance companies by reducing their financial risks in the case of accident.

Thus, insurance does not seem to facilitate the combination of innovation and information acquisition. However, insurance covers part of the cost of damages in the event of an accident. Should the agent go bankrupt after an accident, the insurance company could at least cover some of the financial costs. This may be interpreted as a precautionary measure to protect people and the environment.

7 Conclusion

The most common approach to irreversible investment under uncertainty consists in determining whether the optimal decision is to invest today or to invest tomorrow (Henry, 1974, Epstein, 1980, Dixit and Pindyck, 1994). However, in the race for new technologies, entrepreneurs may not be willing to delay their investment. They have to decide how much they should invest in these new activities today, even if they do not have enough scientific knowledge about the risks of the project for people's health and the environment. Spending some money today to acquire information about the future risks of a project enables entrepreneurs to withdraw from a project if they consider it too risky.

In this paper, we have analysed the investment and information acquisition decisions of an agent who cannot delay her/his investment in a new activity which could potentially put people's health and the environment in danger. We have argued that an agent always invests in the project unless its cost exceeds its direct benefit. However, s/he may decide to invest in the project without doing any research about the potential risks of her/his activity. The agent does look for information when s/he expects that the information s/he will acquire will reach a certain degree of precision, and therefore be reliable.

Moreover, we have found that subjective factors - her/his prior beliefs about the dangerousness of the project, the expected precision of the information and her/his perceived probability of an accident - influence her/his decisions. Subjectivity has a clear effect on the investment decision. Indeed, a higher confidence in the precision and reliability of the information and in the success of the project (i.e., when the agent believes that the "worst state" is unlikely to occur or when s/he expects a low probability of accident) leads the agent to make a larger investment and motivates her/him to start the project. On the other hand, the prior beliefs and anticipations of an agent concerning the reliability of the information have an ambiguous effect on the agent's decision to acquire information or not. Only the expected information precision is an incentive to acquire information. Moreover, the cost of information acquisition C indirectly affects the agent's level of investment, but it directly decreases the incentive to acquire information, and in the worse case, to start the project.

In addition, we have discussed the effect of the reduction of uncertainty in the future on the level of investment. We have shown that this decision highly depends on the impact of the investment on the expected benefit in the worst state and on how precise and accurate the agent perceives the information to be. Indeed, if the agent thinks that a higher investment will decrease her/his loss and that his information is precise enough to be useful, s/he will always make a larger investment with information acquisition than without. In compliance with the environmental policies' guidelines, such as the Precautionary Principle or the Polluter-Payer Principle, we have proposed some means of combining innovation and information acquisition. Our model suggests that increasing the expected information precision or decreasing the cost of information acquisition would help to achieve this goal. We then discussed the significance of the evaluation of the research team's performance level and of the States' subsidies, which would enable agents to innovate while taking precautionary measures.

Finally, we have examined the significance of insurance, a risk sharing tools that is widely used by entrepreneurs. We have found that insurance is conducive to higher levels of investment in new activities. However, getting an insurance is not always an optimal decision for the agent. Forcing potential investors to subscribe to insurance plans might discourage them from innovating. Moreover, insurance may have a negative effect on the information acquisition decision, by leading the agent to always continue her/his project in the future and to perceive information acquisition as a useless exercise. Thus, forcing an agent to subscribe to an insurance can decrease her/his probability of combining innovation and information acquisition. However, by reducing the cost of damages in the event of an accident and by covering part of the financial damages, an insurance plan contributes to protecting entrepreneurs, people and the environment. Thus, a new question arises: should insurance be made compulsory in a situation of scientific uncertainty?

Appendix

Proof of Proposition 1

Part *(i)* of Proposition 1

At period 1, for $I \geq 0$, the agent

chooses to continue, i.e., $x = 1$, if:

$$V_1^{NR}(0, I) < V_1^{NR}(1, I) \quad \text{i.e.} \quad E(\theta) < \frac{R(I)}{K(I)};$$

chooses to stop, i.e., $x = 0$, if:

$$V_1^{NR}(1, I) < V_1^{NR}(0, I) \quad \text{i.e.} \quad \frac{R(I)}{K(I)} < E(\theta);$$

is indifferent between stopping and continuing the project, i.e., $x \in \{0, 1\}$, if:

$$V_1^{NR}(1, I) = V_1^{NR}(0, I) \quad \text{i.e.} \quad \frac{R(I)}{K(I)} = E(\theta).$$

Part *(ii)* of Proposition 1

Similar to the proof of part *(i)* of Proposition 1, thus omitted.

■

Proof of Lemma 1

Since $\theta^L < \theta^H$, $0 \leq p_0 \leq 1$ and $\frac{1}{2} < f < 1$, we obtain that:

$$E(\theta|l) - E(\theta) = \frac{(1-p_0)p_0(\theta^H - \theta^L)(1-2f)}{(1-p_0)f + p_0(1-f)} \leq 0$$

and

$$E(\theta) - E(\theta|h) = \frac{(1-p_0)p_0(\theta^L - \theta^H)(2f-1)}{p_0 f + (1-p_0)(1-f)} \leq 0.$$

Thus, $E(\theta|l) \leq E(\theta) \leq E(\theta|h)$.

■

Proof of Lemma 2

We first study the concavity of $V_0^R(1,1,I)$ and $V_0^{NR}(1,I)$: We differentiate twice times $V_0^R(1,1,I)$ and $V_0^{NR}(1,I)$ with respect to I, we obtain:

$$\frac{\partial^2 V_0^R(1,1,I)}{\partial I^2} = \frac{\partial^2 V_0^{NR}(1,I)}{\partial I^2} = R''(I) - E(\theta)K''(I)$$

which is negative because R and K are concave. Thus $V_0^R(1,1,I)$ and $V_0^{NR}(1,I)$ are concave.

Part *(i)* of Lemma 2

If for all $I > 0$ we have $-I + R(I) - E(\theta)K(I) \leq 0$ then $V_0^R(1,1,I)$ and $V_0^{NR}(1,I)$ are negative. So it is never profitable for the agent to invest. The agent's best response is then not to engage her/him in the project.

Moreover, for $I \geq 0$ we have $V_0^R(1,1,I) > V_0^{NR}(1,I)$. So the agent's best response is not to acquire information.

Hence, the agent decides not to invest and not to do any research, i.e. $I_2^* = C_2^* = 0$.

Part *(ii)* of Lemma 2

If there exists $I_2 > 0$ such that $-I_2 + R(I_2) - E(\theta)K(I_2) > 0$ then $V_0^R(1,1,I_2)$ may be positive or negative while $V_0^{NR}(1,I_2)$ is positive.

If $V_0^R(1,1,I_2)$ and $V_0^{NR}(1,I_2)$ are both positive. There exists a solution to the maximization of both $V_0^R(1,1,I)$ and $V_0^{NR}(1,I)$ with respect to I. The first order conditions characterized the agent's investment:

$$\frac{\partial V_0^R(1,1,I)}{\partial I} = \frac{\partial V_0^{NR}(1,I)}{\partial I} = 0 \Leftrightarrow R'(I) - E(\theta)K'(I) = 1. \tag{14}$$

For $I \geq 0$, we compare $V_0^R(1,1,I)$ and $V_0^{NR}(1,I)$, we get: $V_0^R(1,1,I) < V_0^{NR}(1,I)$. So, the agent's best response is to not acquire information.
Overall, the agent does not do any research, $C_2^* = 0$, but s/he invests in the project I_2^* characterized by equation (14).

If $V_0^R(1,1,I_2)$ is negative then it is never profitable for the agent to invest in the project when s/he planned to be informed at period 1. On the other hand, when s/he is uninformed it is profitable for her/him to invest. S/He then maximizes her/his expected profit $V_0^{NR}(1,I)$, and s/he chooses the level of investment which is characterized by equation (14).
Then, for $I \geq 0$, we compare $V_0^R(1,1,I)$ and $V_0^{NR}(1,I)$, we get: $V_0^R(1,1,I) < V_0^{NR}(1,I)$. So, the agent's best response is to not acquire information.
Overall, the agent does not do any research, $C_2^* = 0$, but s/he invests in the project I_2^* characterized by equation (14).
■

Proof of Lemma 3

We first study the concavity of $V_0^R(1,1,I)$: We differentiate twice times $V_0^R(0,1,I)$ with respect to I, we obtain:

$$\frac{\partial^2 V_0^R(0,1,I)}{\partial I^2} = p_0(1-f)(R''(I) - \theta^H K''(I)) + (1-p_0)f(R''(I) - \theta^L K''(I))$$

which is negative because R and K are concave. Thus $V_0^R(0,1,I)$ is concave.

Part *(i)* of Lemma 3

If for all $I > 0$ we have $-I - \bar{C} + p_0(1-f)(R(I) - \theta^H K(I)) + (1-p_0)f(R(I) - \theta^L K(I)) \leq 0$ then $V_0^R(0,1,I)$ is negative. So it is never profitable for the agent to invest. S/He decides not to invest and not to conduct any research, i.e. $I_3^* = C_3^* = 0$.

Part *(ii)* of Lemma 3

If there exists $I_3 > 0$ such that
$-I_3 - \bar{C} + p_0(1-f)(R(I_3) - \theta^H K(I_3)) + (1-p_0)f(R(I_3) - \theta^L K(I_3)) > 0$ then $V_0^R(0,1,I_3)$ is positive.

Since $V_0^R(0,1,I)$ is concave there exists a solution to the maximization of $V_0^R(0,1,I)$ with respect to I. The first order condition characterized the agent's investment:

$$\frac{\partial V_0^R(0,1,I)}{\partial I}=0 \Leftrightarrow p_0(1-f)(R'(I)-\theta^H K'(I))+(1-p_0)f(R'(I)-\theta^L K'(I))=1. \quad (15)$$

So the agent does some research, $C_3^*=\bar{C}$, and s/he invests in the project $I_3^*>0$ which is characterized by equation (15).

■

Proof of Proposition 2

The optimal investment in the project I^* and the optimal cost to acquire information C^* are such that:

- if $V_0^R(0,1,I_3^*) > V_0^{NR}(1,I_2^*)$ and $V_0^R(0,1,I_3^*) > V_0^{NR}(0,I_1^*)$ then $I^*=I_3^*$ and $C^*=\bar{C}$;
- if $V_0^{NR}(1,I_2^*) > V_0^R(0,1,I_3^*)$ and $V_0^{NR}(1,I_2^*) > V_0^{NR}(0,I_1^*)$ then $I^*=I_2^*$ and $C^*=0$;
- if $V_0^{NR}(0,I_1^*) > V_0^R(0,1,I_3^*)$ and $V_0^{NR}(0,I_1^*) > V_0^{NR}(1,I_2^*)$ then $I^*=I_1^*$ and $C^*=0$;

We first compare $V_0^R(0,1,I_3^*)$ and $V_0^{NR}(1,I_2^*)$. We obtain:

$$V_0^R(0,1,I_3^*) > V_0^{NR}(1,I_2^*)$$

which is equivalent to

$$-I_2^*+R\left(I_2^*\right)-E\left(\theta\right)K\left(I_2^*\right) < -I_3^*-\bar{C}+\left(p_0(1-f)+(1-p_0)f\right)\left(R\left(I_3^*\right)-E\left(\theta|l\right)K\left(I_3^*\right)\right) \quad (16)$$

We then compare $V_0^R(0,1,I_3^*)$ and $V_0^{NR}(0,I_1^*)$. We obtain:

$$V_0^R(0,1,I_3^*) > V_0^{NR}(0,I_1^*)$$

which is equivalent to

$$I_3^*+\bar{C} < \left(p_0(1-f)+(1-p_0)f\right)\left(R\left(I_3^*\right)-E\left(\theta|l\right)K\left(I_3^*\right)\right) \quad (17)$$

We compare $V_0^{NR}(1,I_2^*)$ and $V_0^{NR}(0,I_1^*)$. We obtain:

$$V_0^{NR}(1,I_2^*) > V_0^{NR}(0,I_1^*)$$

which is equivalent to

$$I_2^* < R\left(I_2^*\right) - E\left(\theta\right) K\left(I_2^*\right) \tag{18}$$

Overall, if conditions (16) and (17) hold then $I^* = I_1^*$ and $C^* = \bar{C}$. If condition (16) does not hold and condition (18) holds then $I^* = I_2^*$ and $C^* = 0$. Finally, if conditions (17) and (18) do not hold then $I^* = 0$ and $C^* = 0$.

■

Proof of Proposition 3

Part *(i)* of Proposition 3

We differentiate equations (1) and (2) with respect to p_0, respectively. We obtain:

$$\frac{\partial I_2^*}{\partial p_0} = \frac{(\theta^H - \theta^L)K'(I_2^*)}{R''(I_2^*) - E(\theta)K''(I_2^*)}$$

which is negative, and

$$\frac{\partial I_3^*}{\partial p_0} = \frac{-(1-f)(R'(I_3^*) - \theta^H K'(I_3^*)) + f(R'(I_3^*) - \theta^L K'(I_3^*))}{p_0(1-f)(R''(I_3^*) - \theta^H K''(I_3^*)) + (1-p_0)f(R''(I_3^*) - \theta^L K''(I_3^*))}$$

which is negative because as

$$\frac{R'(I_3^*) - \theta^H K'(I_3^*)}{(R'(I_3^*) - \theta^H K'(I_3^*)) + (R'(I_3^*) - \theta^L K'(I_3^*))} < \frac{1}{2}$$

then $-(1-f)(R'(I_3^*) - \theta^H K'(I_3^*)) + f(R'(I_3^*) - \theta^L K'(I_3^*))$ is positive. So, I_2^* and I_3^* are decreasing with p_0.

Part *(ii)* of Proposition 3

We note:

$$g_1(p_0) = -I_3^* - \bar{C} + p_0(1-f)(R(I_3^*) - \theta^H K(I_3^*)) + (1-p_0)f(R(I_3^*) - \theta^L K(I_3^*)),$$

and

$$g_3(p_0) = -I_2^* + R\left(I_2^*\right) - E\left(\theta\right) K\left(I_2^*\right)$$

We differentiate $g_1(p_0)$ and $g_3(p_0)$ with respect to p_0, respectively. We get:

$$g_1'(p_0) = (1-f)\left(R\left(I_3^*\right) - \theta^H K\left(I_3^*\right)\right) - f\left(R\left(I_3^*\right) - \theta^L K\left(I_3^*\right)\right)$$

and

$$g_3'(p_0) = -(\theta^H - \theta^L)K\left(I_2^*\right).$$

Since $\frac{R(I_3^*)-\theta^H K(I_3^*)}{(R(I_3^*)-\theta^H K(I_3^*))+(R(I_3^*)-\theta^L K(I_3^*))} < \frac{1}{2}$then $(1-f)(R(I_3^*)-\theta^H K(I_3^*))-f(R(I_3^*)-\theta^L K(I_3^*))$ is negative. So $g_1(p_0)$ is decreasing with p_0. Moreover, since $\theta^H > \theta^L$, $g_3(p_0)$ is decreasing with p_0.

So, if conditions (3) and (5) hold, a lower p_0 increases the incentive to invest.

Part *(iii)* of Proposition 3

We note:

$$g_2(p_0) = -I_3^* - \bar{C} + p_0(1-f)(R(I_3^*)-\theta^H K(I_3^*)) + (1-p_0)f(R(I_3^*)-\theta^L K(I_3^*)) + I_2^* - R\left(I_2^*\right) + E\left(\theta\right) K\left(I_2^*\right).$$

We differentiate $g_2(p_0)$ with respect to p_0, we obtain:

$$g_2'(p_0) = g_1'(p_0) - g_3'(p_0).$$

The sign is ambiguous because $g_1'(p_0)$ and $g_3'(p_0)$ are both negative. Overall, the effect of p_0 on the information acquisition decision is ambiguous.

■

Proof of Proposition 4

Part *(i)* of Proposition 4

We first differentiate equation (1) with respect to θ^H and θ^L, respectively. We obtain:

$$\frac{\partial I_2^*}{\partial \theta^H} = \frac{p_0 K'(I_2^*)}{p_0(R''(I_2^*) - \theta^H K''(I_2^*)) + (1-p_0)(R''(I_2^*) - \theta^L K''(I_2^*))}$$

and

$$\frac{\partial I_2^*}{\partial \theta^L} = \frac{(1-p_0) K'(I_2^*)}{p_0(R''(I_2^*) - \theta^H K''(I_2^*)) + (1-p_0)(R''(I_2^*) - \theta^L K''(I_2^*))}$$

which are negative. So I_2^* is decreasing with θ^H and θ^L.

We then differentiate equation (2) with respect to θ^H and θ^L, respectively. We obtain:

$$\frac{\partial I_3^*}{\partial \theta^H} = \frac{p_0(1-f)K'(I_3^*)}{(p_0(1-f)(R''(I_3^*) - \theta^H K''(I_3^*)) + (1-p_0)f(R''(I_3^*) - \theta^L K''(I_3^*))}$$

and

$$\frac{\partial I_3^*}{\partial \theta^L} = \frac{(1-p_0)fK'(I_3^*)}{(p_0(1-f)(R''(I_3^*) - \theta^H K''(I_3^*)) + (1-p_0)f(R''(I_3^*) - \theta^L K''(I_3^*))}$$

which are negative. So I_3^* is decreasing with θ^H and θ^L.

Part *(ii)* of Proposition 4

We note:

$$g_1(\theta^H, \theta^L) = -I_3^* - \bar{C} + p_0(1-f)(R(I_3^*) - \theta^H K(I_3^*)) + (1-p_0)f(R(I_3^*) - \theta^L K(I_3^*)),$$

and

$$g_3(\theta^H, \theta^L) = -I_2^* + R\left(I_2^*\right) - E\left(\theta\right) K\left(I_2^*\right)$$

We differentiate $g_1(\theta^H, \theta^L)$ and $g_3(\theta^H, \theta^L)$ with respect to θ^H and θ^L, respectively. We obtain:

$$\frac{\partial g_1(\theta^H, \theta^L)}{\partial \theta^H} = -p_0(1-f)K(I_3^*) \text{ and } \frac{\partial g_1(\theta^H, \theta^L)}{\partial \theta^L} = -(1-p_0)K(I_3^*)$$

and

$$\frac{\partial g_3(\theta^H, \theta^L)}{\partial \theta^H} = -p_0 K(I_2^*) \text{ and } \frac{\partial g_3(\theta^H, \theta^L)}{\partial \theta^L} = -(1-p_0)K(I_2^*)$$

which are negative. $g_1(\theta^H, \theta^L)$ and $g_3(\theta^H, \theta^L)$ are both decreasing with θ^H and θ^L. So a lower θ^H and/or a lower θ^L increase the incentive to invest.

Part *(iii)* of Proposition 4

We note:

$$\begin{aligned} g_2(\theta^H, \theta^L) = \; & -I_3^* - \bar{C} + p_0(1-f)(R(I_3^*) - \theta^H K(I_3^*)) + (1-p_0)f(R(I_3^*) - \theta^L K(I_3^*)) \\ & + I_2^* - R\left(I_2^*\right) + E\left(\theta\right) K\left(I_2^*\right). \end{aligned}$$

We differentiate $g_2(\theta^H, \theta^L)$ with respect to θ^H and θ^L, respectively, we obtain:

$$\frac{\partial g_2(\theta^H, \theta^L)}{\partial \theta^H} = \frac{\partial g_1(\theta^H, \theta^L)}{\partial \theta^H} - \frac{\partial g_3(\theta^H, \theta^L)}{\partial \theta^H} \text{ and } \frac{\partial g_2(\theta^H, \theta^L)}{\partial \theta^L} = \frac{\partial g_1(\theta^H, \theta^L)}{\partial \theta^L} - \frac{\partial g_3(\theta^H, \theta^L)}{\partial \theta^L}.$$

The sign of these equations are ambiguous because $\frac{\partial g_1(\theta^H, \theta^L)}{\partial \theta^H}$, $\frac{\partial g_3(\theta^H, \theta^L)}{\partial \theta^H}$, $\frac{\partial g_1(\theta^H, \theta^L)}{\partial \theta^L}$ and $\frac{\partial g_3(\theta^H, \theta^L)}{\partial \theta^L}$ are negative. So the effects of θ^H and θ^L on the information acquisition decision are ambiguous.

■

Proof of Proposition 5

Part *(i)* of Proposition 5

We differentiate equation (2) with respect to f, we obtain:

$$\frac{\partial I_3^*}{\partial f} = \frac{p_0(R'(I_3^*) - \theta^H K'(I_3^*)) - (1-p_0)(R'(I_3^*) - \theta^L K'(I_3^*))}{p_0(1-f)(R''(I_3^*) - \theta^H K''(I_3^*)) + (1-p_0)f(R''(I_3^*) - \theta^L K''(I_3^*))}.$$

If $R'(I_3^*) - \theta^H K'(I_3^*) \leq 0$ then $\frac{\partial I_3^*}{\partial f}$ is positive. On the other hand, if $R'(I_3^*) - \theta^H K'(I_3^*) > 0$ then:

- if $p_0 < \frac{R'(I_3^*) - \theta^L K'(I_3^*)}{(R'(I_3^*) - \theta^H K'(I_3^*)) + (R'(I_3^*) - \theta^L K'(I_3^*))}$ then $\frac{\partial I_3^*}{\partial f}$ is positive.
- otherwise, it is negative.

Part *(ii)* of Proposition 5

We note:

$$g_1(f) = -I_3^* - \bar{C} + p_0(1-f)(R(I_3^*) - \theta^H K(I_3^*)) + (1-p_0)f(R(I_3^*) - \theta^L K(I_3^*)),$$

and

$$g_2(f) = -I_3^* - \bar{C} + p_0(1-f)(R(I_3^*) - \theta^H K(I_3^*)) + (1-p_0)f(R(I_3^*) - \theta^L K(I_3^*)) + I_2^* - R(I_2^*) + E(\theta) K(I_2^*).$$

We differentiate $g_1(f)$ and $g_2(f)$ with respect to f, respectively. We obtain:

$$\frac{\partial g_1(f)}{\partial f} = \frac{\partial g_2(f)}{\partial f} = -p_0(R(I_3^*) - \theta^H K(I_3^*)) + (1-p_0)(R(I_3^*) - \theta^L K(I_3^*)).$$

If $R(I_3^*) - \theta^H K(I_3^*) \leq 0$ then $\frac{\partial g_1(f)}{\partial f}$ and $\frac{\partial g_2(f)}{\partial f}$ are positive. Otherwise, if $R(I_3^*) - \theta^H K(I_3^*) > 0$ the agent anticipates that s/he will never acquire information because it is always optimal for her/him to continue her/his project.

■

Proof of Proposition 6

We differentiate $V_0^R(0,1,I)$ and $V_0^{NR}(1,I)$ with respect to I, respectively, we get:

$$\frac{\partial V_0^R(0,1,I)}{\partial I} = -1 + p_0(1-f)(R'(I) - \theta^H K'(I)) + (1-p_0)f(R'(I) - \theta^L K'(I)) \quad (19)$$

and

$$\frac{\partial V_0^{NR}(1,I)}{\partial I} = -1 + p_0(R'(I) - \theta^H K'(I)) + (1-p_0)(R'(I) - \theta^L K'(I)). \quad (20)$$

We replace I by I_2^* in equation (19), and I by I_3^* in equation (20). According to equations (1) and (2), we obtain:

$$-\left[p_0 f(R'(I_2^*) - \theta^H K'(I_2^*)) + (1-p_0)(1-f)(R'(I_2^*) - \theta^L K'(I_2^*))\right] \quad (21)$$

and

$$p_0 f(R'(I_3^*) - \theta^H K'(I_3^*)) + (1 - p_0)(1 - f)(R'(I_3^*) - \theta^L K'(I_3^*)).$$

From equations (1) and (2), and Lemma 1, we note that:

$$R'(I_2^*) - \theta^H K'(I_2^*) < R'(I_2^*) - E(\theta)K'(I_2^*) = 1 < R'(I_2^*) - \theta^L K'(I_2^*)$$

and

$$R'(I_3^*)-\theta^H K'(I_3^*) < p_0(1-f)(R'(I_3^*)-\theta^H K'(I_3^*))+(1-p_0)f(R'(I_3^*)-\theta^L K'(I_3^*)) = 1 < R'(I_3^*)-\theta^L K'(I_3^*).$$

Hence, $R'(I_2^*) - \theta^H K'(I_2^*)$ and $R'(I_3^*) - \theta^H K'(I_3^*)$ may be positive, negative or equal to zero while $R'(I_2^*) - \theta^L K'(I_2^*)$ and $R'(I_3^*) - \theta^L K'(I_3^*)$ are always positive.

Part *(i)* of Proposition 6

If $R'(I_2^*) - \theta^H K'(I_2^*) \geq 0$ then the slope of $V_0^R(0, 1, I_2^*)$ is negative implying that $I_3^* < I_2^*$.

If $R'(I_3^*) - \theta^H K'(I_3^*) \geq 0$ then the slope of $V_0^{NR}(1, I_3^*)$ is positive implying that $I_3^* < I_2^*$.

Part *(ii)* of Proposition 6

If $R'(I_2^*) - \theta^H K'(I_2^*) < 0$ then:

- if $\frac{(1-p_0)(R'(I_2^*)-\theta^L K'(I_2^*))}{(1-p_0)(R'(I_2^*)-\theta^L K'(I_2^*))-p_0(R'(I_2^*)-\theta^H K'(I_2^*))} < f$, the slope of $V_0^R(0, 1, I_2^*)$ is positive implying that $I_2^* < I_3^*$;
- Otherwise the slope of $V_0^R(0, 1, I_2^*)$ is negative or null implying that $I_3^* \leq I_2^*$.

If $R'(I_3^*) - \theta^H K'(I_3^*) < 0$ then:

- if $\frac{(1-p_0)(R'(I_3^*)-\theta^L K'(I_3^*))}{(1-p_0)(R'(I_3^*)-\theta^L K'(I_3^*))-p_0(R'(I_3^*)-\theta^H K'(I_3^*))} < f$, the slope of $V_0^{NR}(1, I_3^*)$ is negative implying that $I_2^* < I_3^*$;
- Otherwise the slope of $V_0^{NR}(1, I_3^*)$ is positive or null implying that $I_3^* \leq I_2^*$.

■

Proof of Proposition 7

Similar to the proof of Proposition 1, thus omitted.

■

Proof of Lemma 4

We study the concavity of $V_0^{RI}(1,1,I,P)$ and $V_0^{NRI}(1,I,P)$. We differentiate $V_0^{RI}(1,1,I,P)$ and $V_0^{NRI}(1,I,P)$ with respect to I and P, respectively, we obtain:

$$\frac{\partial V_0^{RI}(1,1,I,P)}{\partial I} = \frac{\partial V_0^{NRI}(1,I,P)}{\partial I} = -1 + R'(I) - E(\theta)\frac{\partial K(I,P)}{\partial I}. \tag{22}$$

and

$$\frac{\partial V_0^{RI}(1,1,I,P)}{\partial P} = \frac{\partial V_0^{NRI}(1,I,P)}{\partial P} = -1 - E(\theta)\frac{\partial K(I,P)}{\partial P}. \tag{23}$$

We differentiate equation (22) with respect to I and equation (23) with respect to P, we obtain:

$$\frac{\partial^2 V_0^{RI}(1,1,I,P)}{\partial I^2} = \frac{\partial^2 V_0^{NRI}(1,I,P)}{\partial I^2} = R''(I) - E(\theta)\frac{\partial^2 K(I,P)}{\partial I^2}$$

and

$$\frac{\partial^2 V_0^{RI}(1,1,I,P)}{\partial P^2} = \frac{\partial^2 V_0^{NRI}(1,I,P)}{\partial P^2} = -E(\theta)\frac{\partial^2 K(I,P)}{\partial P^2}$$

which are negative. Moreover, we differentiate equation (22) with respect to P, we obtain:

$$\frac{\partial^2 V_0^{RI}(1,1,I,P)}{\partial I \partial P} = \frac{\partial^2 V_0^{NRI}(1,I,P)}{\partial I \partial P} = -E(\theta)\frac{\partial^2 K(I,P)}{\partial I \partial P}$$

Hence, the determinant of the Hessian matrix is equal to:

$$\left[R''(I) - E(\theta)\frac{\partial^2 K(I,P)}{\partial I^2}\right]\left[-E(\theta)\frac{\partial^2 K(I,P)}{\partial P^2}\right] - \left[-E(\theta)\frac{\partial^2 K(I,P)}{\partial I \partial P}\right]^2$$

which is positive by assumptions. Thus, $V_0^{RI}(1,1,I,P)$ and $V_0^{NRI}(1,I,P)$ are concave.

There exists a solution to the maximization of $V_0^{RI}(1,1,I,P)$ and $V_0^{NRI}(1,I,P)$ which is characterized by the system of the two first order conditions.

$$\begin{cases} \frac{\partial V_0^{RI}(1,1,I,P)}{\partial I} = \frac{\partial V_0^{NRI}(1,I,P)}{\partial I} = 0 \Leftrightarrow R'(I) - E(\theta)\frac{\partial K(I,P)}{\partial I} = 1 \\ \frac{\partial V_0^{RI}(1,1,I,P)}{\partial I} = \frac{\partial V_0^{NRI}(1,I,P)}{\partial P} = 0 \Leftrightarrow -E(\theta)\frac{\partial K(I,P)}{\partial P} = 1 \end{cases}$$

$$\Rightarrow R'(I) - E(\theta)\left[\frac{\partial K(I,P)}{\partial I} - \frac{\partial K(I,P)}{\partial P}\right] = 0$$

■

Proof of Lemma 5

Similar to the proof of Lemma 2, thus omitted.

■

Proof of Lemma 6

Similar to the proof of Lemma 4, thus omitted.

■

Proof of Lemma 7

Similar to the proof of Lemma 3, thus omitted.

■

Proof of Proposition 8

Similar to the proof of Proposition 2, thus omitted.

■

Proof of Lemma 8

For $P > 0$, we differentiate $V_0^{NRI}(1, I, P)$ with respect to I, we obtain:

$$\frac{\partial V_0^{NRI}(1, I, P)}{\partial I} = -1 + R'(I) - E(\theta)\frac{\partial K(I, P)}{\partial I} \tag{24}$$

We replace I by I_2^* in equation (24). According to equation (1), we obtain:

$$E(\theta)\left(K'(I_2^*) - \frac{\partial K(I_2^*, P)}{\partial I}\right). \tag{25}$$

By assumption, we get that $K'(I_2^*) - \frac{\partial K(I_2^*,P)}{\partial I} > 0$ then equation (25) is positive. So the slope of $V_0^{NRI}(1, I_2^*)$ is positive implying that $I_2^* < I_2^{BR}$.

To compare I_3^* and I_3^{BR}, the proof is similar, thus omitted.

■

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